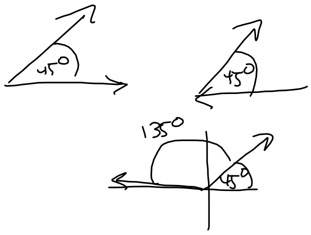
Questions?

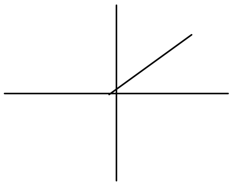
§12.2 # Never mind.

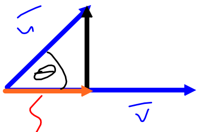
§12.4 #1, 4, 7, 13, 15*, 20, 22, 27, 28, 33, 37, 41, 46

* Picture is nose-to-tail, so you need to think about the angle between the vectors

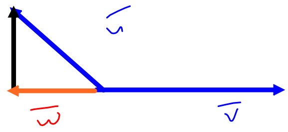
Keenan? wants me to be on the proper screen!



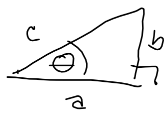




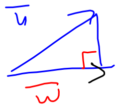
$\text{proj}_{\bar{v}}\bar{u} = \bar{w}$



Thinking as a triangle.



$a = c\cos\theta$



$\|\bar{w}\| = \|\bar{u}\|\cos\theta$

Recall,

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}$$

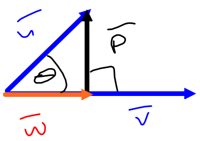
$\text{comp}_{\bar{v}}\bar{u}$ = component of $\bar{u}$ on (in the direction of) $\bar{v}$ is the signed length of $\bar{u}$'s "shadow" projected onto $\bar{v}$

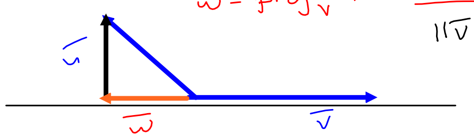
So $\text{comp}_{\bar{v}}\bar{u} = (\cos\theta)(\|\bar{u}\|)$

$$= \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\|\bar{v}\|}\|\bar{u}\| = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|}$$

is a scalar. Now, to get PROJECTION, we multiply this length by unit vector in direction of $\bar{v}$.

$$\bar{w} = \text{proj}_{\bar{v}}\bar{u} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|}\cdot\frac{\bar{v}}{\|\bar{v}\|} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|^2}\bar{v} = \text{proj}_{\bar{v}}\bar{u}$$





$\bar{w} + \bar{p} = \bar{u}$

$\bar{p} = \bar{u} - \bar{w} = \bar{u} - \text{proj}_{\bar{v}}\bar{u}$

$\bar{w} = \text{orth}_{\bar{u}}\bar{v}$

$\text{orth}_{\bar{v}}\bar{u}$ is the component of $\bar{u}$ that's parallel to $\bar{v}$.

$\bar{p} \perp \bar{v}$

$\bar{p}$ is the "part" (component) of $\bar{u}$ that is orthogonal (perpendicular) to $\bar{v}$

The $\bar{p}$ is what Gram-Schmidt gives us.

We decomposed **u** into 2 components. One is parallel to **v** (and therefore redundant) and the other is perpendicular to **v**, which we need to span 2-space.

§12.4 Cross Product.

– Determinant of a matrix

$3x = 7$ $\quad$ $x = \frac{7}{3}$

$A\bar{x} = \bar{c}$

$\frac{A\bar{x}}{A} = \frac{\bar{c}}{A}$ Almost

$A^{-1}(A\bar{x}) = A^{-1}\bar{c}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = A$$

$$|A| = \det(A) = (1)(4) - (3)(2) = 4 - 6 = \boxed{-2 = |A|}$$

$|A| = 0$ means not invertible, so not all $A\bar{x} = \bar{b}$ have solutions.

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 5 \\ 2 & 1 & 7 \end{bmatrix} = A$$

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

Expansion by minors:

I'll expand across the 1^{st} row:

$$1\big((0)(7) - (1)(5)\big) - 2\big((-1)(7) - (2)(5)\big) + 3\big((-1)(1) + (2)(0)\big)$$

$$= -5 + 34 - 3 = 26 = |A|$$

CROSS PRODUCT IS This Determinant

$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2 v_3 - u_3 v_2)\bar{i} - (u_1 v_3 - u_3 v_1)\bar{j} + \underline{(u_1 v_2 - u_2 v_1)}\,\bar{k}$$

$$= \langle u_2 v_3 - u_3 v_2, -(u_1 v_3 - u_3 v_1), u_1 v_2 - u_2 v_1 \rangle$$

$\bar{u} = \langle 1, 2, 3 \rangle$

$\bar{v} = \langle -1, 1, 2 \rangle$

$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{vmatrix} = \langle 1, -(2-(-3)), 3 \rangle = \langle 1, -5, 3 \rangle$$

Computational technique.

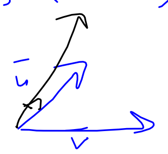
$$\begin{array}{rl} & \langle 1, 2, 3 \rangle, 1, 2 \\ \times & \langle -1, 1, 2 \rangle, -1, 1 \\ \hline & \langle 1, -5, 3 \rangle \end{array}$$

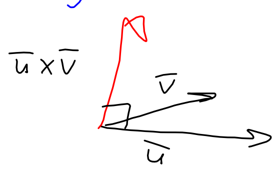
$\langle a, b, c \rangle = \bar{n}$

$\bar{n} \cdot \bar{x} = \langle a, b, c \rangle \cdot \langle x, y, z \rangle$

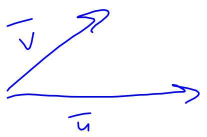
$= ax + by + cz = 7$

FACT $\bar{u} \times \bar{v}$ is a vector, orthogonal to $\bar{u}$ & $\bar{v}$

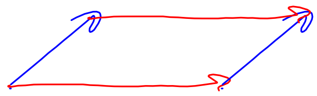




$\bar{u} \times \bar{v}$ is perpendicular to the plane containing $\bar{u}, \bar{v}$.



$\|\bar{u} \times \bar{v}\|$ = area of the parallelogram



$|\bar{u} \cdot (\bar{v} \times \bar{w})|$ = volume of the parallelepiped.

$\boxed{\text{scalar triple product}}$