12.1, 12.2 questions

Vectors, Distance

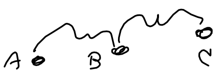
$\bar{v} = \langle v_1, v_2, v_3 \rangle$, $\bar{u} = \langle u_1, u_2, u_3 \rangle$

§12.1

11a $\langle 2, 4, 2 \rangle$

$A(2,4,2)$, ~~$B(3,7,-2)$~~, $C(1,3,3)$

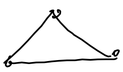
$B(3,7,-2)$




$D(A,B) + D(B,C) = D(A,C)$

See if larger distance is the sum of the two smaller distances.

It will generally be less



$D(A,B) = \sqrt{1^2 + 3^2 + 4^2} = \sqrt{26}$

$D(A,C) = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$D(B,C) = \sqrt{2^2 + 4^2 + 5^2} = \sqrt{45}$

S12.3 #'s 13, 14, 16, 19, 33, 34, 40, 42, 45, 46

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Direction cosines (unimportant)

Orthogonal Projection

Gram-Schmidt orthogonalization

$\bar{i} = \langle 1,0,0\rangle$

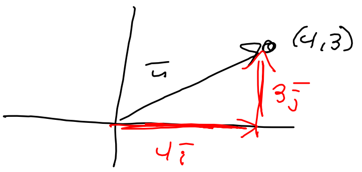
$\bar{j} = \langle 0,1,0\rangle$

$\bar{k} = \langle 0,0,1\rangle$

Canonical Basis for $\mathbb{R}^3$.

$$\langle 1,2,-7\rangle = \langle 1,0,0\rangle + \langle 0,2,0\rangle + \langle 0,0,-7\rangle$$
$$= 1\bar{i} + 2\bar{j} - 7\bar{k}$$

We can decompose any vector in $\mathbb{R}^3$ into a linear combination of $\bar{i}, \bar{j}, \bar{k}$.




$\bar{u} = \langle 4,3\rangle$

$= 4\bar{i} + 3\bar{j}$

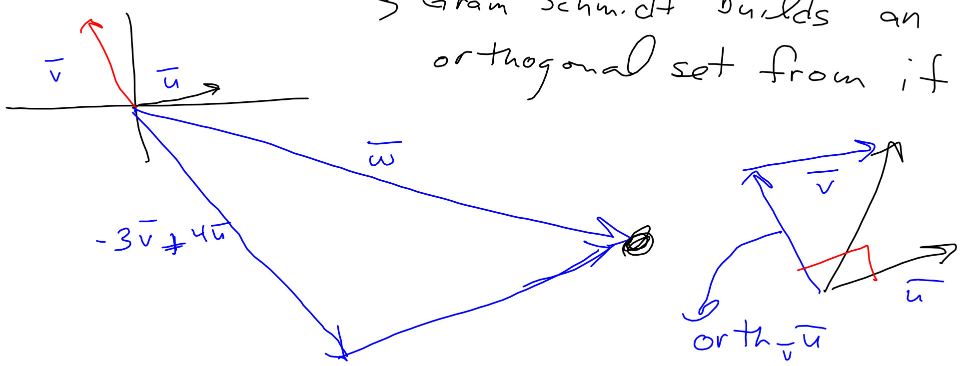
$\bar{i} = \langle 1,0\rangle, \bar{j} = \langle 0,1\rangle$

Basis: Can build any vector as a linear combo on the vectors in the basis, and is a minimal set that can do so.

Spans

$\{\langle 1,2\rangle, \langle -3,4\rangle\}$ spans $\mathbb{R}^2$

Gram Schmidt builds an orthogonal set from it



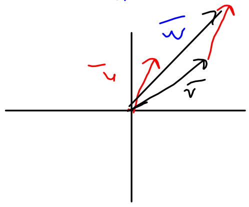
vector addition

$\bar{u} + \bar{v} = \langle u_1+v_1, u_2+v_2, u_3+v_3 \rangle$

Nose-to-tail, Take diagonal.

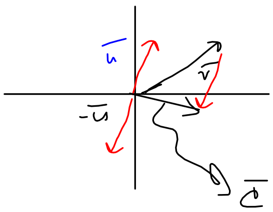
$\bar{u} = \langle 1,2,3 \rangle, \quad \bar{v} = \langle 3,2,-1 \rangle$

$\bar{u} = \langle 1,2 \rangle, \quad \bar{v} = \langle 3,2 \rangle \qquad \bar{u} + \bar{v} = \langle 4,4 \rangle = \bar{w}$




$\bar{w} = \bar{u} + \bar{v}$

$\bar{v} - \bar{u} = \bar{v} + (-\bar{u})$

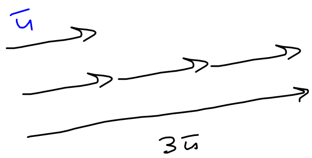



$\bar{v} - \bar{u} = \langle 2,0 \rangle$

My pictures suck!

$\bar{c} = \bar{v} - \bar{u}$

Scalar multiples of vectors.



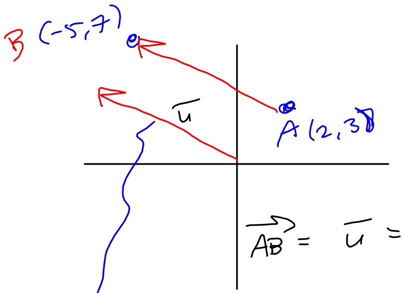

$3\bar{u} = 3\langle u_1, u_2, u_3 \rangle$

$= \langle 3u_1, 3u_2, 3u_3 \rangle$

$\bar{u} = \langle 1,2,3 \rangle \Rightarrow$

$3\bar{u} = \langle 3,6,9 \rangle$

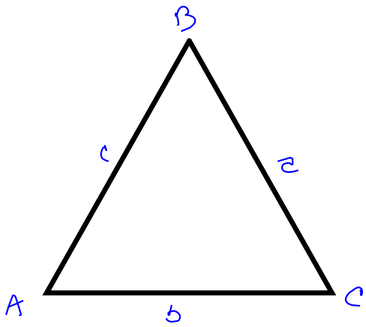
Parallel vectors are multiples of one another.

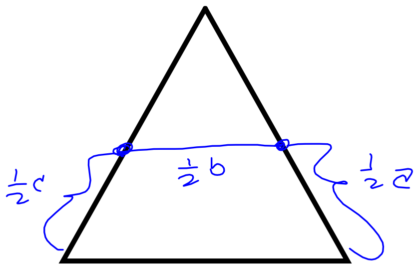


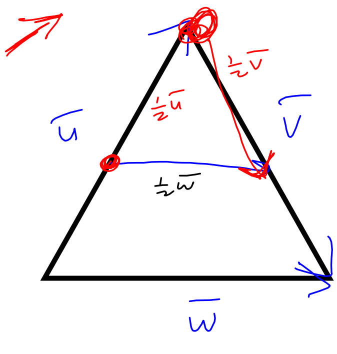

The directed line segment $\overrightarrow{AB}$ is the vector $\bar{u}$

$\overrightarrow{AB} = \bar{u} = \langle -5-2, 7-3 \rangle = \langle -7, 4 \rangle$

THE representative of all directed line segments of same length & direction as $\overrightarrow{AB}$



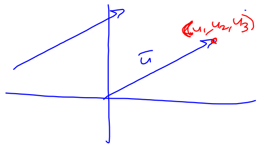




$\bar{u} + \bar{v} = \overline{w}$

$\frac{1}{2}\bar{u} + \frac{1}{2}\bar{v} = \frac{1}{2}(\bar{u} + \bar{v}) = \frac{1}{2}\overline{w}$

Length of a vector comes from Pythagoras




$\|\bar{u}\|$ = magnitude of $\bar{u}$, a vector.

$|u|$ = absolute value of the NUMBER $u$

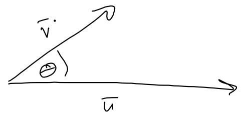
$$\|\bar{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

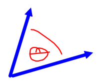
Dot Product.

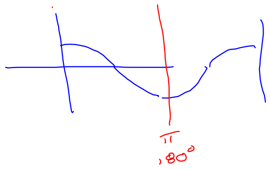
$$\bar{u} \bullet \bar{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

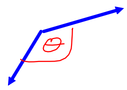
$$\langle 1,2,3\rangle \bullet \langle 3,-1,1\rangle = (1)(3) + (2)\cdot(-1) + (3)(1)$$
$$= 3 - 2 + 3 = -2$$

Angle Between Vectors

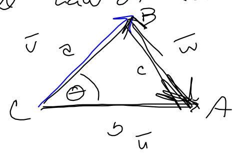


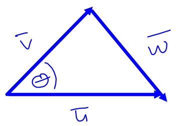




Recall Law of cosines




$$c^2 = a^2 + b^2 - 2ab\cos\theta$$
$$\|\bar{w}\|^2 = \|\bar{v}\|^2 + \|\bar{u}\|^2 - 2\|\bar{v}\|\|\bar{u}\|\cos\theta$$




$\bar{u} = \bar{v} + \bar{w}$

$\bar{u} - \bar{v} = \bar{w}$

$$\|\bar{u} - \bar{v}\|^2 = \|u\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

NOTE: $\|\bar{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$

$$= \sqrt{u_1 u_1 + u_2 u_2 + u_3 u_3}$$
$$= \sqrt{\bar{u} \bullet \bar{u}}$$

$$\langle u_1, u_2, u_3\rangle \bullet \langle u_1, u_2, u_3\rangle = u_1^2 + u_2^2 + u_3^2 = \|\bar{u}\|^2$$

$$(\bar{u} - \bar{v}) \bullet (\bar{u} - \bar{v}) = \bar{u} \bullet \bar{u} + \bar{v} \bullet \bar{v} - 2\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$\bar{u} \bullet \bar{u} - \bar{u} \bullet \bar{v} - \bar{v} \bullet \bar{u} + \bar{v} \bullet \bar{v} = \bar{u} \bullet \bar{u} + \bar{v} \bullet \bar{v} + \sim\sim$$

$$- \quad \underline{u_1 v_1 + u_2 v_2 + u_3 v_3} - \underline{(v_1 u_1 + v_2 u_2 + v_3 u_3)}$$

$$\bar{u} \bullet \bar{v} = \bar{v} \bullet \bar{u}$$

Keenan sez I can't do arithmetic!

So we have

$$-\cancel{2}\,\bar{u} \bullet \bar{v} = -\cancel{2}\|\bar{u}\|\|\bar{v}\|\cos\theta$$

$$\boxed{\frac{\bar{u} \bullet \bar{v}}{\|\bar{u}\|\|\bar{v}\|} = \cos\theta}$$

Cheat Sheet

That's enuff

3pm

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