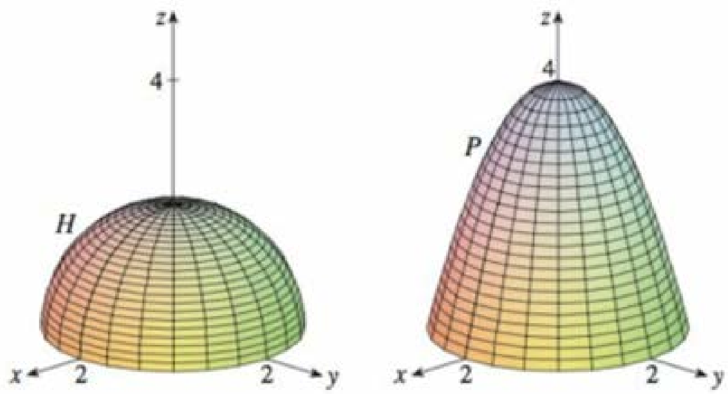
1. A hemisphere H and a portion P of a paraboloid are shown. Suppose $\mathbf{F}$ is a vector field on $\mathbb{R}^3$ whose components have continuous partial derivatives. Explain why

$$\iint_H \text{curl } \mathbf{F} \cdot d\mathbf{S} = \iint_P \text{curl } \mathbf{F} \cdot d\mathbf{S}$$



2–6 Use Stokes' Theorem to evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$.

2. $\mathbf{F}(x, y, z) = 2y \cos z \, \mathbf{i} + e^x \sin z \, \mathbf{j} + xe^y \, \mathbf{k}$,
S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geqslant 0$, oriented upward

3. $\mathbf{F}(x, y, z) = x^2z^2 \, \mathbf{i} + y^2z^2 \, \mathbf{j} + xyz \, \mathbf{k}$,
S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$, oriented upward

4. $\mathbf{F}(x, y, z) = x^2y^3z \, \mathbf{i} + \sin(xyz) \, \mathbf{j} + xyz \, \mathbf{k}$,
S is the part of the cone $y^2 = x^2 + z^2$ that lies between the planes $y = 0$ and $y = 3$, oriented in the direction of the positive y-axis

7–10 Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. In each case C is oriented counterclockwise as viewed from above.

7. $\mathbf{F}(x, y, z) = (x + y^2) \, \mathbf{i} + (y + z^2) \, \mathbf{j} + (z + x^2) \, \mathbf{k}$,
C is the triangle with vertices (1, 0, 0), (0, 1, 0), and (0, 0, 1)

13–15 Verify that Stokes' Theorem is true for the given vector field $\mathbf{F}$ and surface S.

13. $\mathbf{F}(x, y, z) = y^2 \, \mathbf{i} + x \, \mathbf{j} + z^2 \, \mathbf{k}$,
S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$, oriented upward

15. $\mathbf{F}(x, y, z) = y\,\mathbf{i} + z\,\mathbf{j} + x\,\mathbf{k}$,
S is the hemisphere $x^2 + y^2 + z^2 = 1$, $y \geq 0$, oriented in the direction of the positive y-axis

16. Let C be a simple closed smooth curve that lies in the plane $x + y + z = 1$. Show that the line integral

$$\int_C z\,dx - 2x\,dy + 3y\,dz$$

depends only on the area of the region enclosed by C and not on the shape of C or its location in the plane.

17. A particle moves along line segments from the origin to the points $(1, 0, 0)$, $(1, 2, 1)$, $(0, 2, 1)$, and back to the origin under the influence of the force field

$$\mathbf{F}(x, y, z) = z^2\,\mathbf{i} + 2xy\,\mathbf{j} + 4y^2\,\mathbf{k}$$

Find the work done.

18. Evaluate

$$\int_C (y + \sin x)\,dx + (z^2 + \cos y)\,dy + x^3\,dz$$

where C is the curve $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$, $0 \leq t \leq 2\pi$. [*Hint:* Observe that C lies on the surface $z = 2xy$.]

19. If S is a sphere and $\mathbf{F}$ satisfies the hypotheses of Stokes' Theorem, show that $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = 0$.

20. Suppose S and C satisfy the hypotheses of Stokes' Theorem and f, g have continuous second-order partial derivatives. Use Exercises 24 and 26 in Section 16.5 to show the following.

(a) $\int_C (f\,\nabla g) \cdot d\mathbf{r} = \iint_S (\nabla f \times \nabla g) \cdot d\mathbf{S}$

(b) $\int_C (f\,\nabla f) \cdot d\mathbf{r} = 0$

(c) $\int_C (f\,\nabla g + g\nabla f) \cdot d\mathbf{r} = 0$