Section 15.3 Double/Iterated Integrals over General Regions

S 15.3 #s 1, 8, 15, 19, 20, 31, 48, 53 from Handout.

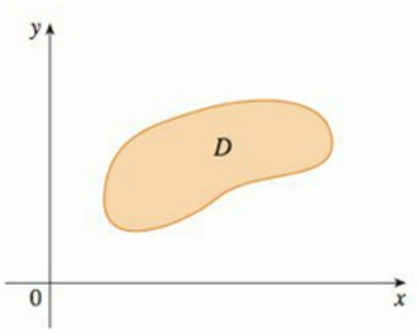


FIGURE 1

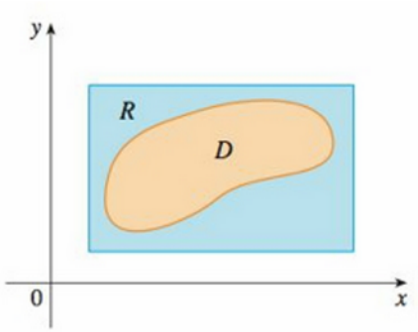
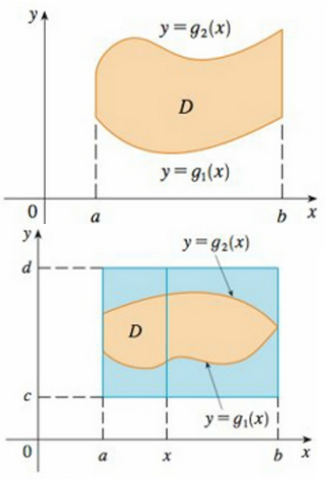


FIGURE 2

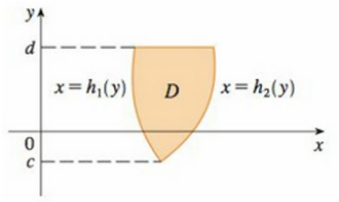
$$F(x, y) = \begin{cases} f(x, y) & \text{if } (x, y) \text{ is in } D \\ 0 & \text{if } (x, y) \text{ is in } R \text{ but not in } D \end{cases}$$



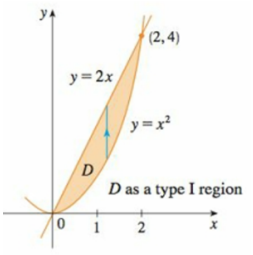
TYPE I

$$\iint_D f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx$$

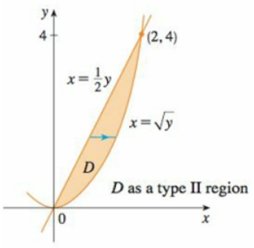
TYPE II

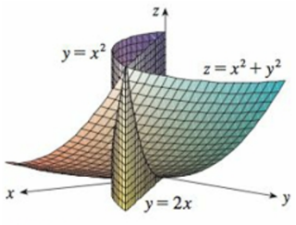


$$\iint_D f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy$$

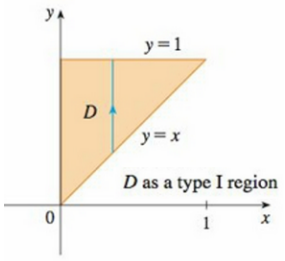


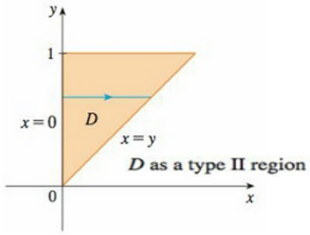
EXAMPLE 2 Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.



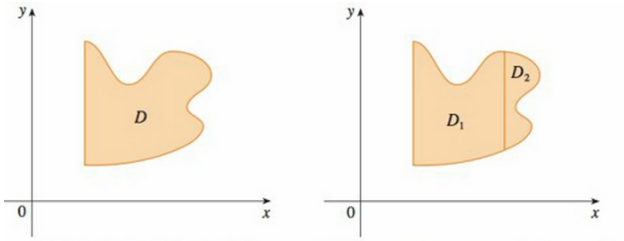


EXAMPLE 5 Evaluate the iterated integral $\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx$.





We can partition the domain D in order to make things work.



(a) D is neither type I nor type II. (b) $D = D_1 \cup D_2$, D_1 is type I, D_2 is type II.

$$\iint_D 1 \, dA = A(D)$$

If $m \leq f(x, y) \leq M$ for all (x, y) in D, then $mA(D) \leq \iint_D f(x, y) \, dA \leq MA(D)$