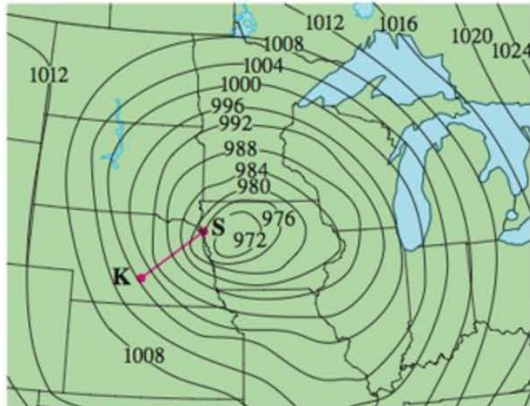
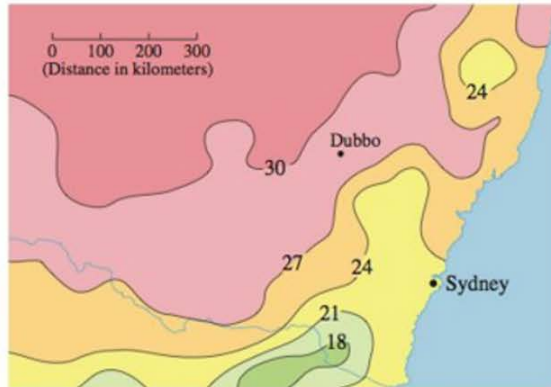


- 1.** Level curves for barometric pressure (in millibars) are shown for 6:00 AM on November 10, 1998. A deep low with pressure 972 mb is moving over northeast Iowa. The distance along the red line from K (Kearney, Nebraska) to S (Sioux City, Iowa) is 300 km. Estimate the value of the directional derivative of the pressure function at Kearney in the direction of Sioux City. What are the units of the directional derivative?



From *Meteorology Today*, 8E by C. Donald Ahrens (2007 Thomson Brooks/Cole).

- 2.** The contour map shows the average maximum temperature for November 2004 (in °C). Estimate the value of the directional derivative of this temperature function at Dubbo, New South Wales, in the direction of Sydney. What are the units?



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- 3.** A table of values for the wind-chill index $W = f(T, v)$ is given in Exercise 3 on page 924. Use the table to estimate the value of $D_u f(-20, 30)$, where $\mathbf{u} = (\mathbf{i} + \mathbf{j})/\sqrt{2}$.

4–6 Find the directional derivative of f at the given point in the direction indicated by the angle θ .

- 4.** $f(x, y) = x^2y^3 - y^4$, $(2, 1)$, $\theta = \pi/4$
5. $f(x, y) = ye^{-x}$, $(0, 4)$, $\theta = 2\pi/3$
6. $f(x, y) = x \sin(xy)$, $(2, 0)$, $\theta = \pi/3$

7–10

- (a) Find the gradient of f .
 (b) Evaluate the gradient at the point P .
 (c) Find the rate of change of f at P in the direction of the vector \mathbf{u} .

7. $f(x, y) = \sin(2x + 3y)$, $P(-6, 4)$, $\mathbf{u} = \frac{1}{2}(\sqrt{3}\mathbf{i} - \mathbf{j})$

8. $f(x, y) = y^2/x$, $P(1, 2)$, $\mathbf{u} = \frac{1}{3}(2\mathbf{i} + \sqrt{5}\mathbf{j})$

9. $f(x, y, z) = xe^{2yz}$, $P(3, 0, 2)$, $\mathbf{u} = \langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \rangle$

10. $f(x, y, z) = \sqrt{x + yz}$, $P(1, 3, 1)$, $\mathbf{u} = \langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \rangle$

11–17 Find the directional derivative of the function at the given point in the direction of the vector \mathbf{v} .

11. $f(x, y) = 1 + 2x\sqrt{y}$, $(3, 4)$, $\mathbf{v} = \langle 4, -3 \rangle$

12. $f(x, y) = \ln(x^2 + y^2)$, $(2, 1)$, $\mathbf{v} = \langle -1, 2 \rangle$

13. $g(p, q) = p^4 - p^2q^3$, $(2, 1)$, $\mathbf{v} = \mathbf{i} + 3\mathbf{j}$

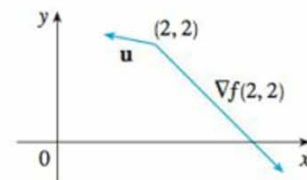
14. $g(r, s) = \tan^{-1}(rs)$, $(1, 2)$, $\mathbf{v} = 5\mathbf{i} + 10\mathbf{j}$

15. $f(x, y, z) = xe^y + ye^z + ze^x$, $(0, 0, 0)$, $\mathbf{v} = \langle 5, 1, -2 \rangle$

16. $f(x, y, z) = \sqrt{xyz}$, $(3, 2, 6)$, $\mathbf{v} = \langle -1, -2, 2 \rangle$

17. $g(x, y, z) = (x + 2y + 3z)^{3/2}$, $(1, 1, 2)$, $\mathbf{v} = 2\mathbf{j} - \mathbf{k}$

- 18.** Use the figure to estimate $D_u f(2, 2)$.



- 19.** Find the directional derivative of $f(x, y) = \sqrt{xy}$ at $P(2, 8)$ in the direction of $Q(5, 4)$.

- 20.** Find the directional derivative of $f(x, y, z) = xy + yz + zx$ at $P(1, -1, 3)$ in the direction of $Q(2, 4, 5)$.

21–26 Find the maximum rate of change of f at the given point and the direction in which it occurs.

21. $f(x, y) = y^2/x$, $(2, 4)$

22. $f(p, q) = qe^{-p} + pe^{-q}$, $(0, 0)$

23. $f(x, y) = \sin(xy)$, $(1, 0)$

24. $f(x, y, z) = (x + y)/z$, $(1, 1, -1)$

25. $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$, $(3, 6, -2)$

26. $f(x, y, z) = \tan(x + 2y + 3z)$, $(-5, 1, 1)$

27. (a) Show that a differentiable function f decreases most rapidly at \mathbf{x} in the direction opposite to the gradient vector, that is, in the direction of $-\nabla f(\mathbf{x})$.
 (b) Use the result of part (a) to find the direction in which the function $f(x, y) = x^4y - x^2y^3$ decreases fastest at the point $(2, -3)$.

28. Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

29. Find all points at which the direction of fastest change of the function $f(x, y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

30. Near a buoy, the depth of a lake at the point with coordinates (x, y) is $z = 200 + 0.02x^2 - 0.001y^3$, where x, y , and z are measured in meters. A fisherman in a small boat starts at the point $(80, 60)$ and moves toward the buoy, which is located at $(0, 0)$. Is the water under the boat getting deeper or shallower when he departs? Explain.

31. The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point $(1, 2, 2)$ is 120° .

- (a) Find the rate of change of T at $(1, 2, 2)$ in the direction toward the point $(2, 1, 3)$.
 (b) Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points toward the origin.

.....
toward the origin.

32. The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$$

where T is measured in $^\circ\text{C}$ and x, y, z in meters.

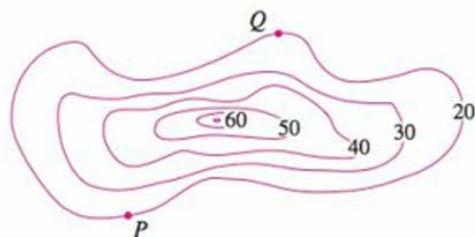
- (a) Find the rate of change of temperature at the point $P(2, -1, 2)$ in the direction toward the point $(3, -3, 3)$.
 (b) In which direction does the temperature increase fastest at P ?
 (c) Find the maximum rate of increase at P .
33. Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$.
- (a) Find the rate of change of the potential at $P(3, 4, 5)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.
 (b) In which direction does V change most rapidly at P ?
 (c) What is the maximum rate of change at P ?

34. Suppose you are climbing a hill whose shape is given by the equation $z = 1000 - 0.005x^2 - 0.01y^2$, where x, y , and z are measured in meters, and you are standing at a point with coordinates $(60, 40, 966)$. The positive x -axis points east and the positive y -axis points north.

- (a) If you walk due south, will you start to ascend or descend? At what rate?
 (b) If you walk northwest, will you start to ascend or descend? At what rate?
 (c) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?

35. Let f be a function of two variables that has continuous partial derivatives and consider the points $A(1, 3)$, $B(3, 3)$, $C(1, 7)$, and $D(6, 15)$. The directional derivative of f at A in the direction of the vector \overrightarrow{AB} is 3 and the directional derivative at A in the direction of \overrightarrow{AC} is 26. Find the directional derivative of f at A in the direction of the vector \overrightarrow{AD} .

36. For the given contour map draw the curves of steepest ascent starting at P and at Q .



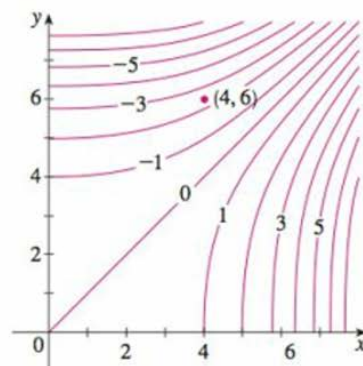
37. Show that the operation of taking the gradient of a function has the given property. Assume that u and v are differentiable functions of x and y and that a, b are constants.

(a) $\nabla(au + bv) = a\nabla u + b\nabla v$ (b) $\nabla(uv) = u\nabla v + v\nabla u$

(c) $\nabla\left(\frac{u}{v}\right) = \frac{v\nabla u - u\nabla v}{v^2}$ (d) $\nabla u^n = nu^{n-1}\nabla u$

38. Sketch the gradient vector $\nabla f(4, 6)$ for the function f whose

level curves are shown. Explain how you chose the direction and length of this vector.



39–44 Find equations of (a) the tangent plane and (b) the normal line to the given surface at the specified point.

39. $2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10$, $(3, 3, 5)$

40. $y = x^2 - z^2$, $(4, 7, 3)$

41. $x^2 - 2y^2 + z^2 + yz = 2$, $(2, 1, -1)$

42. $x - z = 4 \arctan(yz)$, $(1 + \pi, 1, 1)$

43. $z + 1 = xe^y \cos z$, $(1, 0, 0)$

44. $yz = \ln(x+z)$, $(0, 0, 1)$