4. The wave heights h in the open sea depend on the speed v of the wind and the length of time t that the wind has been blowing at that speed. Values of the function $h = f(v, t)$ are recorded in feet in the following table.
You needn't reproduce the table if you attach the exercises as a cover sheet.

Duration (hours)

Wind speed (knots) v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

(a) What are the meanings of the partial derivatives $\partial h/\partial v$ and $\partial h/\partial t$?
(b) Estimate the values of $f_v(40, 15)$ and $f_t(40, 15)$. What are the practical interpretations of these values?
(c) What appears to be the value of the following limit?

$$\lim_{t\to\infty} \frac{\partial h}{\partial t}$$

13–14 Find f_x and f_y and graph f, f_x, and f_y with domains and viewpoints that enable you to see the relationships between them.

13. $f(x, y) = x^2 + y^2 + x^2y$ **14.** $f(x, y) = xe^{-x^2-y^2}$ #14 NA

21. $f(x, y) = \dfrac{x - y}{x + y}$ **26.** $f(x, t) = \arctan(x\sqrt{t})$

29. $f(x, y, z) = xz - 5x^2y^3z^4$ **30.** $f(x, y, z) = x\sin(y - z)$

49–50 Find $\partial z/\partial x$ and $\partial z/\partial y$.

49. (a) $z = f(x) + g(y)$ (b) $z = f(x + y)$

50. (a) $z = f(x)g(y)$ (b) $z = f(xy)$
(c) $z = f(x/y)$ #49 NA

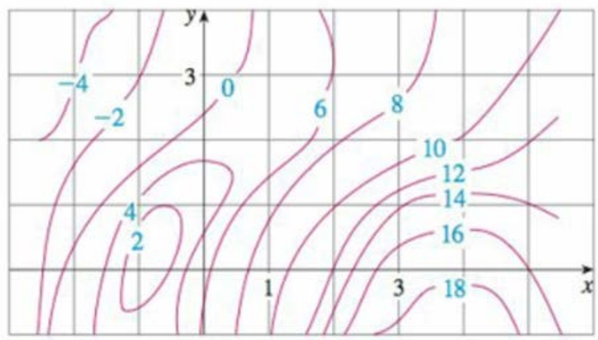
57–60 Verify that the conclusion of Clairaut's Theorem holds, that is, $u_{xy} = u_{yx}$. #57 NA

57. $u = x\sin(x + 2y)$ **59.** $u = \ln\sqrt{x^2 + y^2}$

78. Show that the Cobb-Douglas production function $P = bL^{\alpha}K^{\beta}$ satisfies the equation

$$L\frac{\partial P}{\partial L} + K\frac{\partial P}{\partial K} = (\alpha + \beta)P$$

10. A contour map is given for a function f. Use it to estimate $f_x(2, 1)$ and $f_y(2, 1)$.



11. If $f(x, y) = 16 - 4x^2 - y^2$, find $f_x(1, 2)$ and $f_y(1, 2)$ and interpret these numbers as slopes. Illustrate with either hand-drawn sketches or computer plots.

15–38 Find the first partial derivatives of the function.

15. $f(x, y) = y^5 - 3xy$ **18.** $f(x, t) = \sqrt{x}\ln t$

45–48 Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$.

45. $x^2 + y^2 + z^2 = 3xyz$ **47.** $x - z = \arctan(yz)$

#45 NA

51–56 Find all the second partial derivatives.

52. $f(x, y) = \sin^2(mx + ny)$ **54.** $v = \dfrac{xy}{x - y}$

#54 NA

53. $w = \sqrt{u^2 + v^2}$ **56.** $v = e^{xe^y}$

71. Verify that the function $u = e^{-\alpha^2k^2t}\sin kx$ is a solution of the *heat conduction equation* $u_t = \alpha^2u_{xx}$.

82. The gas law for a fixed mass m of an ideal gas at absolute temperature T, pressure P, and volume V is $PV = mRT$, where R is the gas constant. Show that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1$$

83. For the ideal gas of Exercise 82, show that

$$T\frac{\partial P}{\partial T}\frac{\partial V}{\partial T} = mR$$