MAT 203 Test 2 – Fall, 2018 Name________________________

70 Points Covers Chapter 14 NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

1. (5 pts) Find and graph the domain of $f(x,y) = \sqrt{x} + \sqrt{16 - x^2 - 4y^2}$.

2. (Bonus 5 pts) Use the fact that $x^2 \geq x^2 - y^2$ to prove that $\lim_{(x,y)\to(0,0)} \frac{\sqrt{x^2 - y^2}}{x} = 0$.

3. Find the first partials f_x and f_y for...

   a. (5 pts) $f(x,y) = \sqrt{2x^2 - xy - y^2}$

   b. (5 pts) $f(x,y) = \int_{\cos(x^2)}^{y^3} \frac{t^4 \tan(t^2)}{t^2 + 5}\, dt$

4. (5 pts) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the equation $xyz - x\cos(x^2 y) = z^2$

5. Let $f(x,y) = e^{x^2 - y^2}$.

   a. (5 pts) Find an equation of the tangent plane to f at the point $(1,-1,1)$.

   b. (5 pts) Use your previous answer to approximate $f(1.2,-0.9)$.

   c. (5 pts) Find the actual value of $f(1.2,-0.9)$.

   d. (5 pts) Find Δz for the change in z from $f(1,-1) = 1$ to $f(1.2,-0.9)$

   e. (5 pts) Find the differential approximation $dz \approx \Delta z$. You may calculate this, directly, or just use previous work and a subtraction.

6. Let $f(x,y) = \frac{y^2}{x}$.

   a. (5 pts) What is the gradient at the point $P(1,2,4)$?

   b. (5 pts) Find the directional derivative $D_{\bar{u}}$ in the direction of $\bar{u} = \langle 2, \sqrt{5} \rangle$.

7. Find the shortest distance between the plane $x + 2y + z = 4$ and the point $P(1,-2,3)$ in two ways:

   a. (5 pts) Use 1st- and/or 2nd- derivative test.

   b. (5 pts) Use earlier skills from Chapter 12.

8. (5 pts) Find the point on the plane in #7 that is closest to $P(1,-2,3)$.

Bonus: Answer up to 3 of the following for up to 15 bonus points.

1. (5 pts) (Line segment) Write the equation of the line segment between $A(1,2,3)$ and $B(-3,2,1)$.

2. (5 pts) Consider the object $9x^2 + 4z^2 - 25y = 0$. Show its traces in the planes $x = k, y = k, z = k$ for different choices of k and project those into the $yz-, xz-,$ and $xy-$planes, respectively.

3. (5 pts) Give a verbal description of the statement $\kappa = \left|\frac{d\bar{T}}{ds}\right|$. What is it? What does it mean? What's our shortcut for calculating it, in terms of $\bar{r}(t)$?

Distance Formula in Three Dimensions The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3 Theorem If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: $\quad \text{comp}_{\mathbf{a}}\, \mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

Vector projection of $\mathbf{b}$ onto $\mathbf{a}$: $\quad \text{proj}_{\mathbf{a}}\, \mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}\right) \dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\, \mathbf{a}$

9 Theorem If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ (so $0 \leqslant \theta \leqslant \pi$), then

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$$

2 Parametric equations for a line through the point (x_0, y_0, z_0) and parallel to the direction vector $\langle a, b, c \rangle$ are

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

Elliptic Paraboloid

Arc Length:

$$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\, dt$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$L = \int_a^b |\mathbf{r}'(t)|\, dt$$