MAT 203 Test 1 – Fall, 2018 Name_________________

100 Points Covers Chapters 12, 13 NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

Leave space between problems. No prizes for saving paper, here. Figure this stuff out, and use your smarts to plant trees! Only use one column of work. Don't start a 2nd column to save paper. ALL I WANT ON THIS PAGE IS YOUR NAME.

1. (10 pts) Find parametric equations and a vector equation for the line of intersection between the two planes:
   $P_1:\ x-3y+4z=-7$
   $P_2:\ 2x-5y+7z=-11$

2. Essential concepts for math in 3-D:
   a. (10 pts) (Line) Form the vector $\bar{u}=\overrightarrow{AB}$ and find a vector equation for the line containing the points $A(1,2,3)$ and $B(-3,2,1)$.

   b. (10 pts) (Line segment) Write the equation of the line segment between *A* and *B*.

   c. (10 pts) (Vector Equation of Plane) Form the vector $\bar{v}=\overrightarrow{AC}$, using *A* from part a. and $C(8,-5,4)$. Then write a vector equation for the plane containing *A*, *B* and *C*.

   d. (10 pts) (General Equation of a Plane) Write the general form of your answer to part c. You may safely stop at $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$, rather than going all the way to $ax+by+cz=d$.

   e. (10 pts) (Area of Parallelogram) Find the area of the parallelogram defined by the vectors $\bar{u}$ and $\bar{v}$.

   f. (10 pts) (Volume of a Parallelepiped) Form the vector $\bar{w}=\overrightarrow{AD}$, where $D(-2,-2,1)$ is another point. And find the volume of the parallelepiped defined by the 3 edges, $\bar{u},\bar{v}$, and $\bar{w}$.

3. Distance Problems:
   a. (10 pts) (Distance from a Point to a Line) Find the distance from the point $D(-2,-2,1)$ to the line L: $\bar{r}(t)=\bar{r}_0+t\bar{v}=\langle 3,1,0\rangle+t\langle 1,2,3\rangle$. This is not the same $\bar{v}$ as in #2. For convenience, use the letter *E* for the point corresponding to the initial (position) vector $\bar{r}_0=\langle 3,1,0\rangle$, i.e., $E=(3,1,0)$.

   b. (10 pts) (Distance from a Point to a Plane) Find the distance from the point $D(-2,-2,1)$ to the plane P: $P: 3x-2y+7z=8$

4. (10 pts) Describe and sketch $9x^2+4z^2-25y=0$. Show its traces in the planes $x=k, y=k, z=k$ for different choices of *k* and project those into the $yz-, xz-$, and $xy-$ planes, respectively.

5. (10 pts) Find the Unit Tangent $\bar{T}$, Unit Normal $\bar{N}$ and Unit Binormal $\bar{B}$ in *general*, for some $\bar{r}(t)$.

6. (10 pts) Give a verbal description of the statement $\kappa=\left|\frac{d\bar{T}}{ds}\right|$. What is it? What does it mean? What's our shortcut for calculating it, in terms of $\bar{r}(t)$?

Distance Formula in Three Dimensions The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$\boxed{3}$ **Theorem** If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: $\quad \text{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

Vector projection of $\mathbf{b}$ onto $\mathbf{a}$: $\quad \text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}\right)\dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}$

$\boxed{9}$ **Theorem** If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ (so $0 \leq \theta \leq \pi$), then

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

$\boxed{2}$ Parametric equations for a line through the point (x_0, y_0, z_0) and parallel to the direction vector $\langle a, b, c\rangle$ are

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

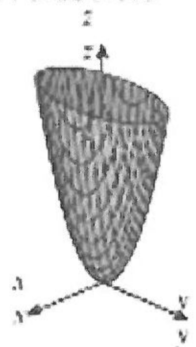
Arc Length:

$$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\,dt$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

$$L = \int_a^b |\mathbf{r}'(t)|\,dt$$

Elliptic Paraboloid



203 T1 Sol'ns FALL '18

1 10 pts

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -7 \\ 2 & -5 & 7 & -11 \end{array}\right]$$

$$\begin{array}{c} R1 \\ -2R1+R2 \end{array}\left[\begin{array}{ccc|c} 1 & -3 & 4 & -7 \\ 0 & 1 & -1 & 3 \end{array}\right]\begin{array}{c} 3R2+R1 \\ R2 \end{array}\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$$x + z = 2 \Longrightarrow x = -z + 2$$
$$y - z = 3 \qquad y = z + 3$$

So, the line is given by

$x = -t+2,\ y = t+3,\ z = t$ in parametric form, which gives rise to the vector equation

$$r(t) = \langle 2, 3, 0\rangle + t\langle -1, 1, 1\rangle$$

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T1

2a 10 pts $A(1,2,3)$, $B(-3,2,1) \longrightarrow$

$\bar{u} = \overrightarrow{AB} = \langle -4, 0, -2 \rangle = \bar{u}$

a vector eq'n of the line containing A & B

is $\boxed{\langle 1,2,3 \rangle + t\langle -4,0,-2 \rangle = \bar{r}(t)}$: $\mathcal{L}$

b 10 pts $\boxed{\bar{r}(t) = (1-t)\langle 1,2,3 \rangle + t\langle -3,2,1 \rangle}$

$\boxed{0 \le t \le 1}$

d 10 pts $C(8,-5,4) \longrightarrow \bar{v} = \overrightarrow{AC} = \langle 7,-7,1 \rangle$

$$\bar{u} \times \bar{v} = \begin{array}{c} \langle -4, 0, -2 \rangle, -4, 0 \\ \langle 7, -7, 1 \rangle, 7, -7 \\ \hline \langle -14, -10, 28 \rangle \end{array}$$

Let $\bar{n} = \langle 7, 5, -14 \rangle$. It's $\perp$ to P.

So $\bar{n} \cdot (\bar{r} - \bar{r}_0) = 0 \longrightarrow$

$7(x-1) + 5(y-2) - 14(z-3) = 0$

is one version of it, using $A = (x_0, y_0, z_0)$

But wait! This is #2d!

203

T1

2c $\bar{r}(s,t) = \bar{r}_0 + s\bar{u} + t\bar{v}$

10 pts

$$\boxed{\bar{r}(s,t) = \langle 1,2,3\rangle + s\langle -4,0,-2\rangle + t\langle 7,-7,1\rangle \quad \forall (s,t) \in \mathbb{R}^2.}$$

2d Previous Page.

2e 10 pts Area of the parallelogram

is $\|\bar{u}\times\bar{v}\| = \|\langle -14,-10,28\rangle\|$

$= \sqrt{14^2+10^2+28^2} = \sqrt{196+100+784}$

$= \sqrt{1080}$

$$\boxed{= 6\sqrt{30}}$$

$2\,|\,1080$
$2\,|\,540$
$2\,|\,270$
$3\,|\,135$
$3\,|\,45$
$3\,|\,15$
5

$28^2 = (14(2))^2$

$= 4(196)$

$= 784$

196

980

203 T1

2f 10 pts Volume of parallepiped is

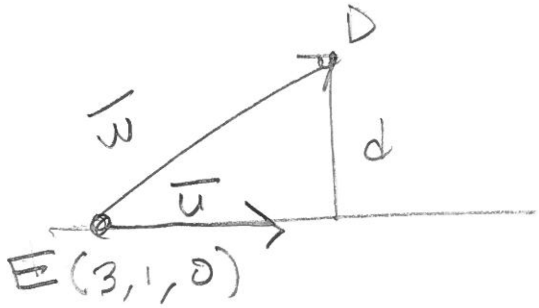
$$V = |\bar{w}\cdot(\bar{u}\times\bar{v})| = |\bar{w}\cdot\langle -14,-10,-28\rangle|$$

$$\bar{w} = \langle -3,-4,-2\rangle \longrightarrow$$

$$V = |\langle -3,-4,-2\rangle\cdot\langle -7,-5,-14\rangle|\,(2)$$

$$= 2|21+20+28| = 2(69) = \boxed{138}$$

3a 10 pts $D(-2,-2,1)$, L: $\bar{r} = \langle 3,1,0\rangle + t\langle 1,2,3\rangle$



$$\bar{r} = \bar{r}_0 + t\bar{u}$$

$$= \langle 3,1,0\rangle + t\langle 1,2,3\rangle$$

$E(3,1,0)$

$$\frac{d}{\|\bar{w}\|} = \sin\theta = \frac{\bar{w}\times\bar{u}}{\|\bar{u}\|\|\bar{w}\|}$$

$$\longrightarrow d = \|\bar{w}\|\sin\theta = \frac{\|\bar{w}\times\bar{u}\|}{\|\bar{u}\|} = \frac{\sqrt{12^2+9^2+7^2}}{\sqrt{1^2+2^2+3^2}}$$

$=$

$\bar{w}$: $\langle -5,-3,1\rangle$, -5, -3

$\bar{u}$: $\langle 1,2,3\rangle$, 1, 2

$\bar{w}\times\bar{u}$ $\langle 12,9,-7\rangle$

203 T1

3a cnt'd

$$= \frac{\sqrt{144+81+49}}{\sqrt{1+4+9}} = \frac{\sqrt{225+49}}{\sqrt{14}} = \sqrt{\frac{274}{14}}$$

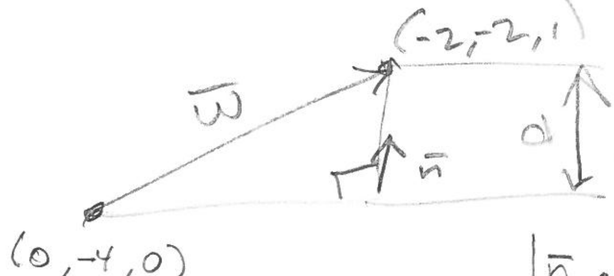
$$\boxed{= \sqrt{\frac{137}{7}}}$$

3b 10 pts Distance from $D(-2,-2,1)$ to

P: $3x - 2y + 7z = 8$

$\bar{n} = \langle 3, -2, 7 \rangle$

$\bar{r}_0 = \langle 0, -4, 0 \rangle$



$\bar{w} = \langle -2, 2, 1 \rangle$

want $|\text{comp}_{\bar{n}} \bar{w}| = \dfrac{|\bar{n} \cdot \bar{w}|}{\|\bar{n}\|} = \dfrac{|-6-4+7|}{\sqrt{3^2+2^2+7^2}}$

$$= \frac{|3|}{\sqrt{62}} \; \boxed{= \frac{3}{\sqrt{62}} = d} = \frac{3\sqrt{62}}{62}$$

203 T1

4 10 pts

$9x^2 + 4z^2 - 25y = 0$

$x = K$:

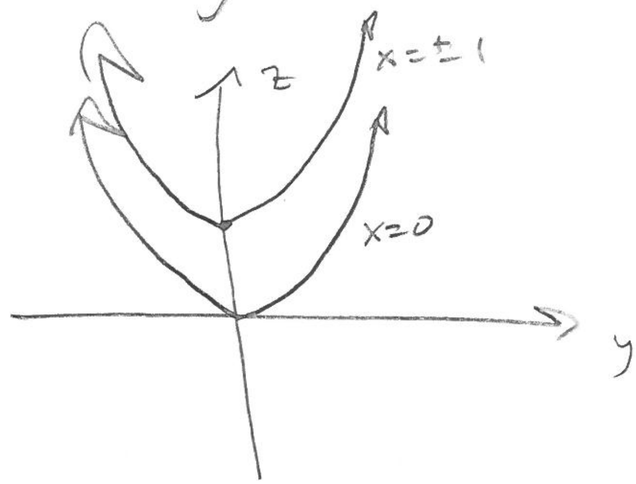
$x = 0$

$25y = 4z^2 + 9(0)^2$

$y = \frac{4}{25} z^2$

$x = \pm 1$

$y = \frac{4}{25} z^2 + 9$



$y = K$:

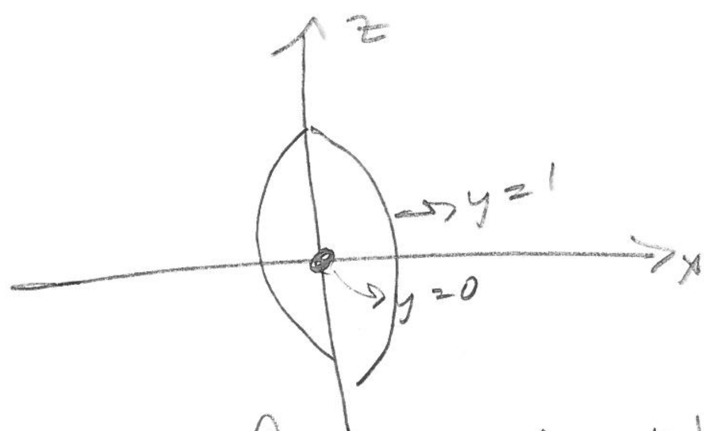
$y=0$ $\quad 9x^2 + 4z^2 = 25(0)$

$x = z = 0$

$y = 1$ $\quad 9x^2 + 4z^2 = 25(1)$

$$\frac{x^2}{\frac{25}{9}} + \frac{z^2}{\frac{25}{4}} = 1$$

$y = -1$ Never



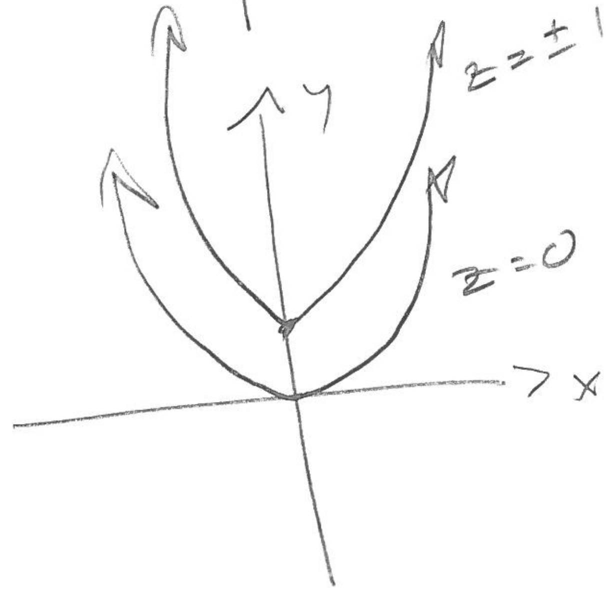
$z = K$:

$z = 0$ $\quad 9x^2 + 4(0)^2 - 25y = 0$

$25y = 9x^2 + 4(0)^2$

$y = \frac{9}{25} x^2 + \frac{4(0)^2}{25}$

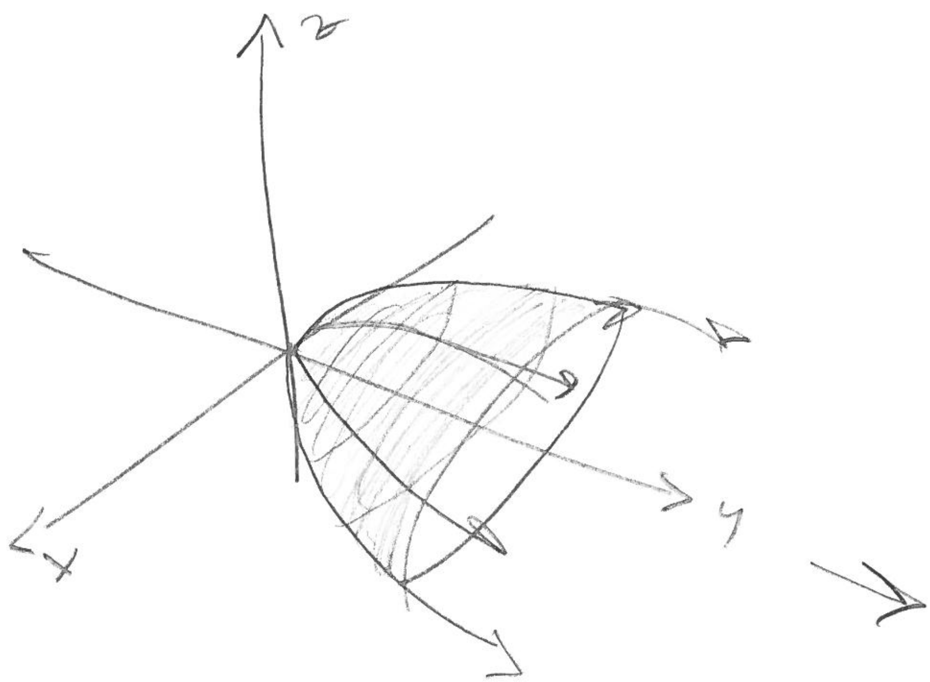
$z = \pm 1$ $\quad y = \frac{9}{25} x^2 + \frac{4}{25}$



(4) cntd By this, we see an elliptical paraboloid, whose cross-sections parallel to the xz-plane are ellipses.

It opens towards the positive y-axis

Cross-sections parallel to yz- and xy-planes are parabolas



203 T1

5 10pts Let $\bar{r}(t)$ be given.

Then $\boxed{\bar{T}(t) = \dfrac{\bar{r}'(t)}{\|\bar{r}'(t)\|}}$, where

$\bar{r}(t) = \langle x(t), y(t), z(t) \rangle$. Then

$$\bar{T}(t) = \frac{\langle x'(t), y'(t), z'(t) \rangle}{\sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}}$$

$$\boxed{\bar{N}(t) = \frac{\bar{T}'(t)}{\|\bar{T}'(t)\|}} \quad \&$$

$$\boxed{\bar{B}(t) = \bar{T}'(t) \times \bar{N}'(t)}$$

6 10pts $\kappa = \left\|\dfrac{d\bar{T}}{ds}\right\|$ is curvature, which is the rate of change of the unit tangent, with respect to arc length. It tells us how sharp the corner is!

$$\kappa = \frac{\|\bar{r}' \times \bar{r}''\|}{\|\bar{r}'\|^3}.$$