2. (10 pts) Evaluate the line integral $\int_C x\,ds$, where S is the line segment joining $(-2,-3)$ to $(3,2)$.

$\bar{r} = (1-t)\langle -2,-3\rangle + t\cdot\langle 3,2\rangle$

$= \langle -2(1-t), -3(1-t)\rangle + \langle 3t, 2t\rangle$

$= \langle -2+2t, -3+3t\rangle + \langle 3t, 2t\rangle$

$= \langle -2+5t, -3+5t\rangle$

$\sqrt{2.25}$

ds = arc length increment

$= \sqrt{x_t^2 + y_t^2}\;dt = \sqrt{5^2+5^2} = 5\sqrt{2}$

$\int_C x\,ds = 5\sqrt{2}\int_0^1 (-2+5t)\,dt = 5\sqrt{2}\left[-2t + \frac{5}{2}t^2\right]_0^1$

$= 5\sqrt{2}\left[-2+\frac{5}{2}\right] = 5\sqrt{2}\cdot\frac{1}{2} = \frac{5\sqrt{2}}{2}$

Dec 5-6:20 PM

3. Let $\overline{F}(x,y,z)=\langle y^2, 2xy+e^{3z}, 3ye^{3z}\rangle = y^2\mathbf{i}+(2xy+3e^{3z})\mathbf{j}+3ye^{3z}\mathbf{k}$.

a. (10 pts) Show that $\overline{F}$ is a conservative field.

b. (10 pts) Find a potential function f such that $\overline{F}=\nabla f$.

§16.5

$\text{curl}\,\overline{F} = \nabla \times \overline{F}$

$\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}$

$\langle y^2, 2xy+e^{3z}, 3ye^{3z} \rangle\ y^2, 2xy+e^{3z}$

$\nabla \cdot f$

∇f

$\langle 3e^{3z}-3e^{3z}, 0-0, 2y-2y \rangle = \overline{0} = \text{curl}\,\overline{F}$

$\Rightarrow$ $\overline{F}$'s conservative.

$\nabla f = \overline{F} \Rightarrow$

$f_x = y^2 \Rightarrow f = y^2x + g(y,z)$

$f_y = 2yx + g_y(y,z) = \underline{2xy+e^{3z}}$

$\Rightarrow g_y(y,z) = e^{3z}$

$\Rightarrow \boxed{g(y,z) = ye^{3z} + h(z)}$

$g_z(y,z) = 3ye^{3z} + h'(z) = 3ye^{3z} \Rightarrow$

$h'(z) = 0$

$\boxed{y^2x + ye^{3z}}$

4. Set up the integral $\int_C x\,dy$ for evaluation, where C is the triangle with vertices $(0,0), (0,4)$, and $(3,0)$ in two ways:

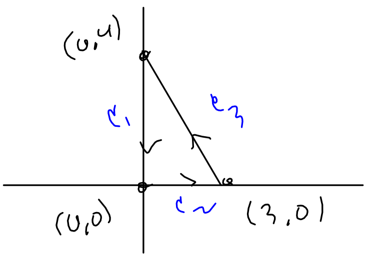
same field for #1. Just wanted to help us see it.

a. (10 pts) Directly, as the sum of 3 line integrals along the edges of C. (Write the 3 integrals, but do not evaluate.)

b. (10 pts) Using Green's Theorem. (Write the iterated integral, but do not evaluate.).

I think the above problem is do-able, but maybe time-consuming. Beware the clock on this one.

5. Let $\overline{F}(x, y, z) = \langle$



$$\int_C x\,dy = \int_{C_1} + \int_{C_2} + \int_{C_3}$$

$$\int_0^4 0\,dy + \int_0^0 x\,dy + \int_0^4 \left(-\frac{3}{4}y+1\right)dy$$

(0,4), (3,0)

$m = \frac{4}{3}$

$y = m(x - x_1) + y_1$

$= -\frac{4}{3}(x - 0) + 4$

$y = -\frac{4}{3}x$

$y = -\frac{4}{3}x + 4$

$y - 4 = -\frac{4}{3}x$

$$\int_0^4 \left(-\frac{3}{4}y + 3\right)dy \qquad x = -\frac{3}{4}(y-4) = -\frac{3}{4}y + 3$$

$$= -\frac{3}{8}y^2 + y\Big]_0^4 = -\frac{3}{8}(16) + \overset{12}{4} = \cancel{-20}$$

$$= -6 + 12 = 6$$

$$\int_0^3 \int_0^{-\frac{4}{3}x+1} x\,dy\,dx = \int_0^3 xy\Big]_0^{-\frac{4}{3}x+4} dx =$$

$$\int_0^3 x\left[-\frac{4}{3}x+4\right]dx = \int_0^3 \left(-\frac{4}{3}x^2+4x\right)dx$$

$$= -\frac{4}{9}x^3 + 2x^2\Big]_0^3 = -\frac{4}{9}(27) + 2(9)$$

$$= -12 + 18 = 6$$