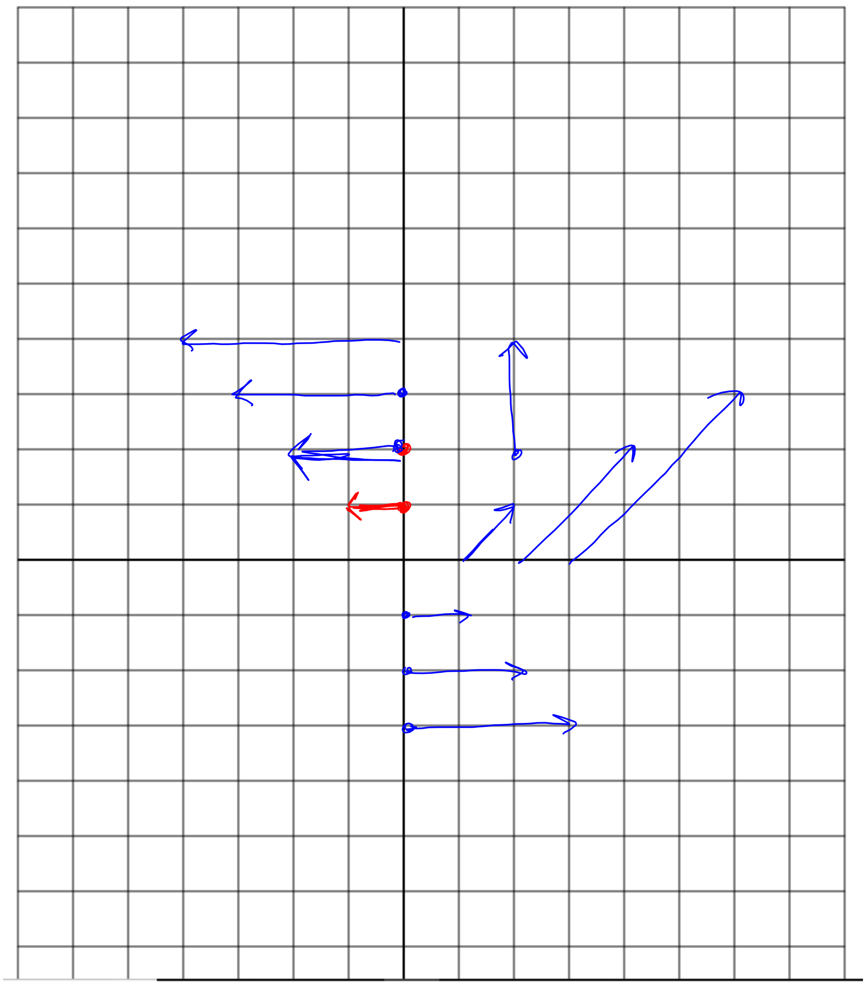
S'16.1 #4 $\vec{F}(x,y) = \langle x-y, x \rangle = (x-y)\vec{i} + x\vec{j}$



$x = 0$

$\langle -y, 0 \rangle$

$(0,1)$ $\langle -1, 0 \rangle$

$(0,2)$ $\langle -2, 0 \rangle$

$y = 0$

$\langle x, x \rangle$

$(0,0)$ $\langle 0,0 \rangle$

$(1,0)$ $\langle 1,1 \rangle$

$(2,0)$ $\langle 2,2 \rangle$

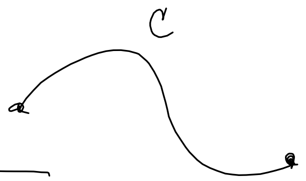
$\langle x-y, x \rangle$

$(2,2)$ $\langle 0,2 \rangle$

We'll play with Maple, a little, on these field plots : Vector Fields, Gradient Fields, etc., but not a big deal for us.

16.2 - Line Integrals

$$\int_C f(x,y)\,ds$$



Recall $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

The increment of arc length

$$= \sqrt{\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dx}\right)^2}\,dx$$

$$= \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\,dx$$

$$\int_C x \sin(y)\,ds$$

C = line segment from (0,3) to (4,6)

Express it as a vector:

$$\bar{r}(t) = (1-t)\langle 0,3\rangle + t\langle 4,6\rangle$$

$$= \langle 0, 3-3t\rangle + \langle 4t, 6t\rangle$$

$$= \boxed{\langle 4t, 3+3t\rangle}$$

$x = x(t) = 4t, \quad y = 3t+3 = y(t)$

sin (3t+3) idiot, mills.

$$\int_0^1 4t\cdot(3t+3)\,ds$$

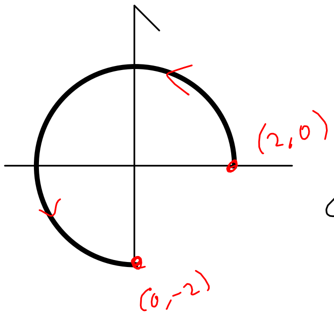
$$= \int_0^1 (12t^2 + 12t)\sqrt{4^2+3^2}\,dt = 5\int_0^1 (12t^2+12t)\,dt$$

$$= 5\left[4t^3 + 6t^2\right]_0^1 = \boxed{50}$$

$$20\int_0^1 t\sin(3t+3)\,dt \text{ is the integral.}$$

$u = t \qquad dv = \sin(3t+3)\,dt$

27. $\mathbf{F}(x, y) = (x - y)\,\mathbf{i} + xy\,\mathbf{j}$,
C is the arc of the circle $x^2 + y^2 = 4$ traversed counter-clockwise from (2, 0) to (0, −2)



$\bar{F}(x,y) = \langle x-y, xy \rangle$

~~Let x be parameter~~

Convert to polar coords

$0 \le \theta \le \frac{3\pi}{2}$

$x = 2\cos\theta,\ y = 2\sin\theta$

$\bar{F}(2\cos\theta, 2\sin\theta) = \langle 2\cos\theta - 2\sin\theta, \boxed{4\sin\theta\cos\theta} \rangle$

$= \bar{r}(\theta)$

$\int_{C_1} \bar{F}\cdot d\bar{r}$

what's arc length increment for this $\langle x(\theta), y(\theta)\rangle$?

$\int_C \bar{F}(\bar{r}(t))\, d\bar{r}$

$d\bar{r} = \bar{r}'(\theta)\, dt = \langle -2\sin\theta - 2\cos\theta, 4\cos^2\theta - 4\sin^2\theta \rangle\, d\theta$

$$= \int_{\theta=0}^{\theta=\frac{3\pi}{2}} \langle 2\cos\theta - 2\sin\theta, 4\sin\theta\cos\theta \rangle \cdot \langle -2\sin\theta - 2\cos\theta, 4\cos^2\theta - 4\sin^2\theta \rangle\, d\theta$$