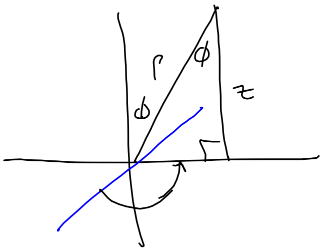
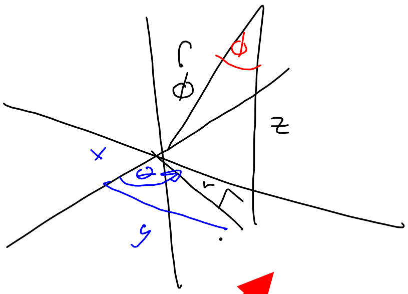
$z = \rho\cos\phi$, $x = r\cos\theta = \rho\sin\phi\cos\theta$

$y = r\sin\theta = \rho\sin\phi\sin\theta$





#15 $\int$ 15.8

Solid below the sphere

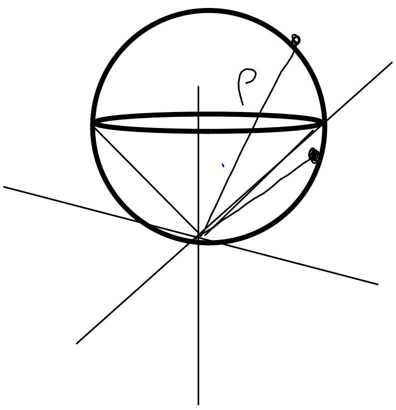
$\boxed{z = x^2+y^2+z^2}$ Above the cone $z = \sqrt{x^2+y^2}$

$x^2+y^2+z^2 - z + \left(\frac{1}{2}\right)^2 = 0 + \frac{1}{4}$

$x^2+y^2+\left(z-\frac{1}{2}\right)^2 = \frac{1}{4}$ center $= \left(0, 0, \frac{1}{2}\right)$

radius $= \frac{1}{2}$

$z = \sqrt{x^2+y^2}$



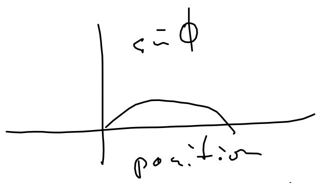
$z = x^2+y^2+z^2 = \sqrt{x^2+y^2}$

when they intersect

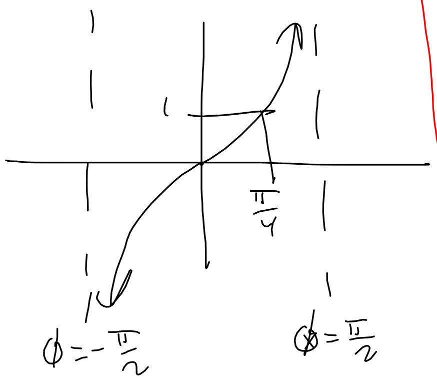
ABOVE THE CONE

$z \geq \sqrt{x^2+y^2}$

$\rho\cos\phi \geq r = \rho\sin\phi$



$\forall \phi \in (0, \pi)$



$\cos\phi \geq \sin\phi$

NOTE $0 \leq \phi \leq \pi$

$\frac{\cos\phi}{\sin\phi} \geq 1$

$\cot\phi \geq 1$

$\tan\phi \leq 1$

$-\frac{\pi}{2} < \phi \leq \frac{\pi}{4}$

$\boxed{0 \leq \phi \leq \frac{\pi}{4}}$

Below the sphere

$z = x^2 + y^2 + z^2$

$\rho\cos\phi = z = x^2 + y^2 + z^2 = \rho^2$

$\rho = \cos\phi$ & want to be below the sphere

so $\rho < \cos\phi$

$\boxed{0 \leq \rho \leq \cos\phi}$

Recall Substitution Rule

$$\int f(x)\,dx$$

what if

$x = g(u)$

$$\int f(g(u))$$

Recall

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du = \int f(g)\,dg$$

$$\int f(g)\,\frac{dg}{dx}\,dx$$

FOR MONDAY, know what the jacobian is.

It's the determinant of

this matrix

$$f(u(x,y),\ g(x,y))$$

Jacobian is

$$\begin{vmatrix} u_x & u_y \\ g_x & g_y \end{vmatrix}$$

$$f(r\cos\theta,\ r\sin\theta)$$

Jacobian is

$$\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$= r\cos^2\theta + r\sin^2\theta = r$!

$$\iint_E f(x,y)\,dA$$

The Jacobian!

$$= \iint_E f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

Change from $\iint_E f(x,y)\,dA$ to

$$\iint_E f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

We went from integrating over a circle in rectangular coords (HARD) to integrating over a polar rectangle in polar coords

$$\{(r,\theta) \mid a \le r \le b,\ c \le \theta \le d\}$$