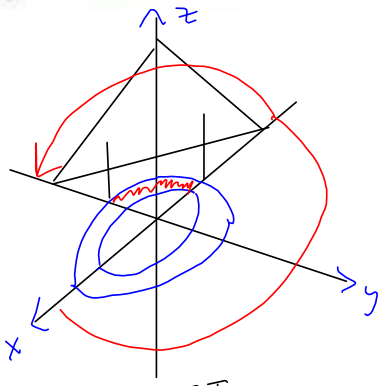
20. Evaluate $\iiint_E x\,dV$, where E is enclosed by the planes $z = 0$ and $z = x + y + 5$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

$r = 3$ $\quad r = 2$



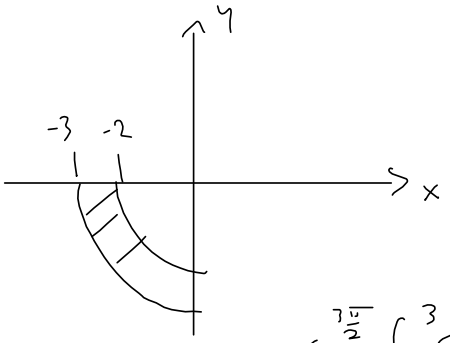
$(0,0,5)$
$(-5,0,0)$
$(0,-5,0)$

$2 \le r \le 3$

$0 \le z \le x+y+5$ inside

$\pi \le \theta \le \frac{3\pi}{2}$

$$\int_{\pi}^{\frac{3\pi}{2}} \int_{2}^{3} \int_{0}^{r\cos\theta + r\sin\theta + 5} r^2\cos\theta \, dz\,dr\,d\theta$$



$$= \int_{\pi}^{\frac{3\pi}{2}} \cos\theta\, d\theta \int_{2}^{3} r^2\, dr \int_{0}^{r} g(r,\theta)$$

Integrate wrt z, THEN FUBINI-IZE IT

$$= \int_{\pi}^{\frac{3\pi}{2}} \int_{2}^{3} \left[z\, r^2 \cos\theta \right]_{0}^{r\cos\theta + r\sin\theta + 5} dr\,d\theta$$

$$= \int_{\pi}^{\frac{3\pi}{2}} \int_{2}^{3} r^3\cos^2\theta \, dr\,d\theta + \int_{\pi}^{\frac{3\pi}{2}} \int_{2}^{3} r^3 \sin\theta\cos\theta \, dr\,d\theta$$

$$\int_{\pi}^{\frac{3\pi}{2}} \int_{2}^{3} 5r^2\cos\theta \, dr\,d\theta$$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

Scratch $\left(r\cos\theta + r\sin\theta + 5\right) r^2\cos\theta$

$= r^3\cos^2\theta + r^3\sin\theta\cos\theta + 5r^2\cos\theta$

$\int u\,du$ situation $= \frac{u^2}{2} + C$

$$= \int_{\pi}^{\frac{3\pi}{2}} \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta \int_2^3 r^3 dr + \int_{\pi}^{\frac{3\pi}{2}} \sin\theta\cos\theta\, d\theta \int_2^3 r^3 dr$$

$$+ 5\int_{\pi}^{\frac{3\pi}{2}} \cos\theta\, d\theta \int_2^3 r^2 dr$$

$u = \sin\theta,\ du = \cos\theta\, d\theta$

$\int u\, du = \frac{1}{2}u^2 + C$

$$\left[\frac{r^4}{4}\right]_2^3 \left(\left[\frac{1}{2}\theta\right]_{\pi}^{\frac{3\pi}{2}} + \left[\frac{1}{4}\sin(2\theta)\right]_{\pi}^{\frac{3\pi}{2}}\right) + \left[\frac{1}{2}\sin^2\theta\right]_{\pi}^{\frac{3\pi}{2}} \left[\frac{1}{4}r^4\right]_2^3$$

$$+ \left[5\sin\theta\right]_{\pi}^{\frac{3\pi}{2}} \left[\frac{1}{3}r^3\right]_2^3$$

$$\left(\frac{81}{4} - \frac{16}{4}\right)\left[\left(\frac{3\pi}{4} - \frac{\pi}{2}\right) + (0 - 0)\right] + \left(\frac{1}{2}\right)\left(\frac{81 - 16}{4}\right)$$

$$+ \left[-5 - 0\right]\left[9 - \frac{8}{3}\right] = \left(\frac{65}{4}\right)\left(\frac{\pi}{4}\right) + \frac{65}{8} + (-5)\left(\frac{19}{3}\right)$$

$$= \frac{65\pi + 130}{16} \cdot \frac{3}{3} - \frac{95}{3} \cdot \frac{16}{16}$$

$\cos(u)\, du = \cos(2\theta) \cdot 2\, d\theta$

$$= \frac{195\pi + 390 - 1520}{48}$$

$$= \frac{195\pi - 1130}{48}$$

$$\begin{array}{r} 2\,950 \\ 475 \\ 95 \\ \hline 520 \\ -\ 390 \\ \hline 130 \end{array}$$