(7) $\int_0^2 \int_0^1 (2x+y)^8 \, dx \, dy$

$\frac{1}{2}\int 2(2x+11)^8 \, dx$ $u = 2x+11$, $du = 2dx$, $\frac{du}{2} = dx$

$= \int u^8 \frac{du}{2}$

$= \frac{1}{2} \cdot \frac{1}{9} u^9 + C$

$= \int_0^2 \frac{1}{2}\left[\frac{(2x+y)^9}{9}\right]_{x=0}^{1} dy$

$= \frac{1}{18}\int_0^2 \left[(2(1)+y)^9 - (2(0)+y)^9\right] dy = \boxed{\frac{1}{18}\int_0^2 \left((2+y)^9 - y^9\right) dy = \frac{1}{180}\left[(y+2)^{10} - y^{10}\right]_0^2}$

$= \frac{1}{180}\left[4^{10} - 2^{10} - (2^{10} - 0^{10})\right]$

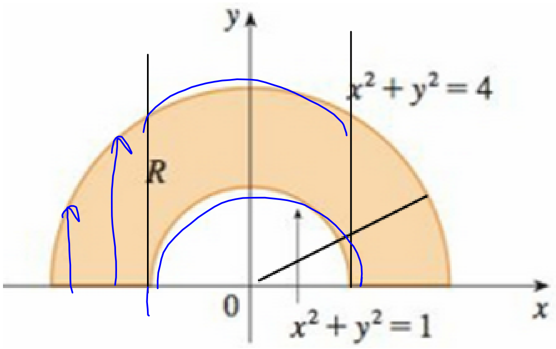
$= \frac{4^{10}}{180}$

$\frac{y^{10}}{180} = \frac{1048576}{180} = \frac{261,632}{45} = 5814.0\overline{4}$

$\frac{1}{2}\int (2x+y)^8 \, 2dx = \frac{1}{2}\left[\frac{(2x+y)^9}{9}\right] + C$

$\int (2x+y)^8 \, dx$ $u = 2x+y$, $du = 2dx$, $dx = \frac{du}{2}$

$= \int u^8 \frac{du}{2} = \frac{1}{2}\int u^8 du = \frac{1}{2} \cdot \frac{u^9}{9} + C = \frac{(2x+y)^9}{18} + C$



$f(x,y) = 3x + 4y^2$

$x^2+y^2=4 \Rightarrow y = \pm\sqrt{4-x^2}$

$= +\sqrt{4-x^2}$ by picture

$$\int_{-2}^{-1}\int_{0}^{\sqrt{4-x^2}} dy\,dx + \int_{-1}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} dy\,dx$$

$$+ \int_{1}^{2}\int_{0}^{\sqrt{4-x^2}} dy\,dx$$ in Rectangular coords.

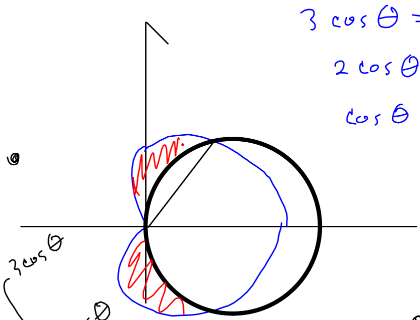
It's difficult.

18. The region inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 3\cos\theta$

$3\cos\theta = 1+\cos\theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$



Isaac $D = 2\int_{0}^{\frac{\pi}{3}} \int_{3\cos\theta}^{1+\cos\theta}$

outer − inner

$$\text{Area} = \int_{\frac{\pi}{3}}^{\pi} \int_{3\cos\theta}^{1+\cos\theta} r\,dr\,d\theta = \int_{\frac{\pi}{3}}^{\pi} \left[\frac{r^2}{2}\right]_{3\cos\theta}^{1+\cos\theta} d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\pi} \left((\cos\theta+1)^2 - 9\cos^2\theta\right) d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\pi} \left(\cos^2\theta + 2\cos\theta + 1 - 9\cos^2\theta\right) d\theta$$

$$= \frac{1}{2}\left[\int_{\frac{\pi}{3}}^{\pi} -8\cos^2\theta\,d\theta + \int_{\frac{\pi}{3}}^{\pi} (2\cos\theta + 1)\,d\theta\right] = \frac{1}{2}\left[A + B\right]$$

$$\Rightarrow A = -\frac{8}{2}\int_{\frac{\pi}{3}}^{\pi} (1+\cos(2\theta))\,d\theta = -4\left[\int_{\frac{\pi}{3}}^{\pi} d\theta + \frac{1}{2}\int_{\frac{\pi}{3}}^{\pi} \cos(2\theta)\cdot 2\,d\theta\right]$$

$\sqrt{3}$, 2, −1

$$= -4\theta\Big]_{\frac{\pi}{3}}^{\pi} - 2\sin(2\theta)\Big]_{\frac{\pi}{3}}^{\pi} = -4\pi + \frac{4\pi}{3} \boxed{-2\sin\frac{2\pi}{3}} = \boxed{-\frac{8\pi}{3} + \sqrt{3} = A}$$

$-2\sin(2\pi) - (-2\sin\frac{2\pi}{3}) = +\sqrt{3}$

$$B = 2\sin\theta + \theta\Big]_{\frac{\pi}{3}}^{\pi} = 2\sin\pi + \pi - \left[2\sin\frac{\pi}{3} + \frac{\pi}{3}\right]$$

$$= \boxed{\frac{2\pi}{3} - \sqrt{3} = B}$$

$$\frac{1}{2}\left[A + B\right] = \frac{1}{2}\left[-\frac{8\pi}{3} + \sqrt{3} + \frac{2\pi}{3} - \sqrt{3}\right]$$

$$= \frac{1}{2}\left[-2\pi - \cancel{2\sqrt{3}}\right] = \boxed{-\pi \cancel{-\sqrt{3}}}$$