Minimize the distance from $P(1,2,3)$
to the plane $x+2y-z=5$: $\mathcal{P}$

$z = x+2y-5$

Let (x,y,z) be a point in $\mathcal{P}$

Then the distance to $P(1,2,3)$ is

$$D = \sqrt{(x-1)^2+(y-2)^2+(z-3)^2}$$

$\sqrt{\text{arg}}$ is increasing function of Arg.

So to minimize D, we can just minimize D^2

$$D^2 = G(x,y) = (x-1)^2+(y-2)^2+(x+2y-5-3)^2$$

$$= (x-1)^2+(y-1)^2+(x+2y-8)^2$$

$$G_x = 2(x-1) + 2(x+2y-8) \overset{SET}{=} 0$$

$$\Rightarrow 2x - 2 + 2x + 4y - 16 = 4x + 4y - 18 \Rightarrow$$

$$4x = 18 - 4y \Rightarrow \boxed{x = \frac{9}{2} - y}$$

$$G_y = 2(y-1) + 2(x+2y-8)(2)$$

$$= 2y - 2 + 4x + 8y - 32$$

$$= 10y + 4x - 34 = 10y + 4\left(\frac{9}{2} - y\right) - 34$$

$$= 10y + 18 - 4y - 34 = 6y - 16 \overset{SET}{=} 0$$

$$6y = 16$$

$$y = \frac{16}{6} = \boxed{\frac{8}{3} = y}$$

$$\Rightarrow x = \frac{9}{2} - \frac{8}{3} = \frac{27-16}{6} = \boxed{\frac{11}{6} = x}$$

The idea is plug in $x, y, \& z$

$$D = \sqrt{(x-1)^2 + (y-2)^2 + (x+2y-5-3)^2}$$

$$= \sqrt{\left(\frac{11}{6} - 1\right)^2 + \left(\frac{8}{3} - 2\right)^2 + \left(\frac{11}{6} + \frac{2(8)}{3} - 8\right)^2}$$

$$= \sqrt{\left(\frac{5}{6}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-5}{6}\right)^2}$$

$$= \sqrt{\frac{25+25}{36} + \frac{4}{9} \cdot \frac{4}{4}} = \sqrt{\frac{66}{36}} = \sqrt{\frac{11}{6}}$$

Closest Point:

$$z = x+2y-5 = \frac{11}{6} + 2\left(\frac{8}{3}\right) - 5$$

$$= \frac{11+32-30}{6} = \boxed{\frac{13}{6} = z}$$

$$(x,y,z) = \boxed{\left(\frac{11}{6}, \frac{8}{3}, \frac{13}{6}\right)}$$

→ closest point

plug in to find distance.

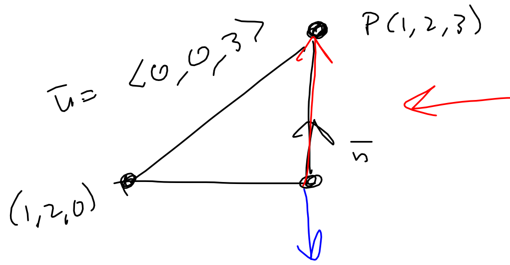
$x+2y-z=5$

$1+2(2)-0=5$

$(1,2,0)$

$\bar{n} = \langle 1,2,-1\rangle$

OR....



$$\text{Dist} = \text{comp}_{\bar{n}}\bar{u} = \frac{|\bar{u}\cdot\bar{n}|}{\|\bar{n}\|} = \frac{9}{\sqrt{6}}$$

So that distance in the (opposite) direction of $\bar{n}$

$$\pm\left(\frac{\bar{n}}{\|\bar{n}\|}\right)\left(\frac{9}{\sqrt{6}}\right) + \langle 1,2,3\rangle = \boxed{\left(\frac{11}{6}, \frac{8}{3}, \frac{13}{6}\right)}$$

Not sure w/o checking.

How would you check?

Plug the result back into

$x+2y-z=5$

See if it works.

IF NEITHER WORKS, kiss your goodbye.