Make your OWN dadgum cheat sheets, henceforth. One Page. 2 Sides.

Re-do Test 2 by working on the test at home and splitting the difference with me.

36. (a) We define the improper integral (over the entire plane $\mathbb{R}^2$)

$$I = \iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\, dA = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dy\, dx$$

$$= \lim_{a\to\infty} \iint_{D_a} e^{-(x^2+y^2)}\, dA \qquad D_a = \{(x,y) \mid \sqrt{x^2+y^2} \le a\}$$

where D_a is the disk with radius a and center the origin. Show that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dA = \pi$$

$$\iint_{D_a} e^{-(x^2+y^2)}\, dA = \int_0^{2\pi}\int_0^a e^{-r^2}\, r\, dr\, d\theta$$

$$= -\frac{1}{2}\int_0^{2\pi}\int_0^a \left(e^{-r^2}\right)(-2r\,dr)\, d\theta$$

$$= -\frac{1}{2}\int_0^{2\pi}\left[e^{-r^2}\right]_0^a d\theta = -\frac{1}{2}\int_0\left[e^{-a^2} - e^0\right] d\theta$$

$$= -\frac{1}{2}\left[e^{-a^2} - 1\right]\theta\Big]_{\theta=0}^{\theta=2\pi}$$

$$= -\frac{1}{2}\left[e^{-a^2} - 1\right]\left[2\pi\right] = -\pi\left[e^{-a^2} - 1\right]$$

$$\xrightarrow{a \to \infty} -\pi\left[0 - 1\right] = \pi \;!$$

Complex Variables.

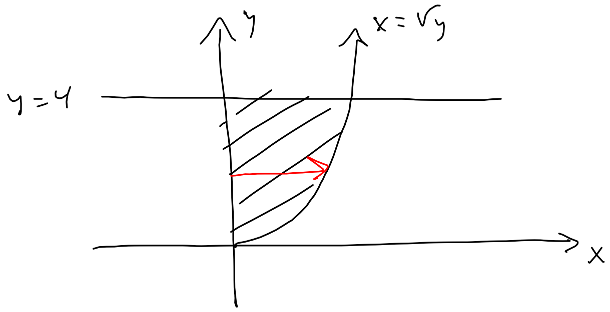
Cauchy Integral Theorem
Residue Theory

$\Big\}$ Complex Integration

§15.3

1 $\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx \, dy = \int_0^y \left[\frac{1}{2}x^2y^2\right]_{x=0}^{x=\sqrt{y}} dy$

$= \int_0^4 \frac{1}{2}y \cdot y^2 \, dy = \cdots = 32 = \frac{1}{8}y^4\Big]_0^4 = \frac{256}{8}$



Claim: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \forall\, n \in \mathbb{N}.$

Proof

$n=1$: $\sum_{k=1}^{1} k = 1 = \frac{1(1+1)}{2} = 1$ ✓

Now suppose it holds for some $n \geq 1$. Then

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ and $\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} k + (n+1)$

$$= \frac{n(n+1)}{2} + n+1 = \frac{n^2+n+2(n+1)}{2} = \frac{n^2+3n+2}{2}$$

$$= \frac{(n+1)(n+2)}{2} = \frac{(n+1)((n+1)+1)}{2},$$ so it holds for $n+1$.

$\therefore$ it holds $\forall\, n \in \mathbb{N}$, by Principle of Mathematical Induction. ∎