Section 15.2 Double/Iterated Integrals

$R = [a, b] \times [c, d]$. S 15.2 #s1, 4, 7, 10, 15, 19, 27, 31, 38 from Handout.

Partial Integration with respect to y:

$\int_c^d f(x, y)\,dy$ means hold x fixed and integrate with respect to y.

$A(x) = \int_c^d f(x, y)\,dy$ is a function of x.

If we now integrate the function A with respect to x from $x = a$ to $x = b$, we get

$$\int_a^b A(x)\,dx = \int_a^b \left[\int_c^d f(x, y)\,dy \right] dx$$ is called an **iterated integral**.

Written in this fashion, it says to integrate w.r.t. y and then integrate w.r.t. x.

EXAMPLE 1 Evaluate the iterated integrals.

(a) $\int_0^3 \int_1^2 x^2 y\,dy\,dx$ (b) $\int_1^2 \int_0^3 x^2 y\,dx\,dy$

Before finishing this example, notice that Fubini says they better both come out the same.

FUBINI'S THEOREM If f is continuous on the rectangle $R = \{(x, y) \mid a \leqslant x \leqslant b,\ c \leqslant y \leqslant d\}$, then

$$\iint_R f(x, y)\,dA = \int_a^b \int_c^d f(x, y)\,dy\,dx = \int_c^d \int_a^b f(x, y)\,dx\,dy$$

More generally, this is true if we assume that f is bounded on R, f is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

If you can factor f thus: $f(x, y) = g(x)h(y)$ then $\iint_R f(x, y)\, dA = \int_a^b g(x)\, dx \int_c^d h(y)\, dy$

Don't expect this situation to arise a whole lot, but when it does, it's very nice.

5. $\int_0^2 \int_0^{\pi/2} x \sin y \, dy\, dx = \int_0^2 x\, dx \int_0^{\frac{\pi}{2}} \sin(y)\, dy$

"Separable"