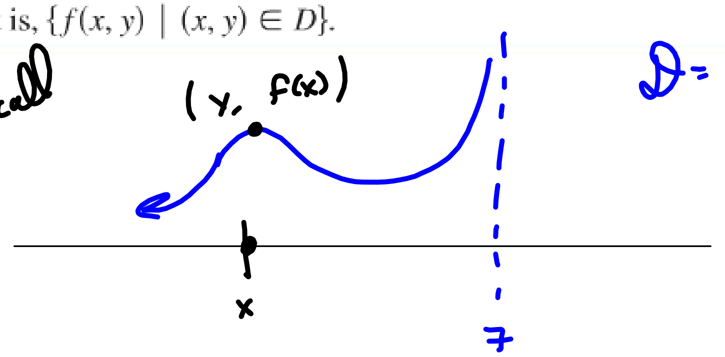
14.1 #s 13, 15, 17-19, 23, 24, 36, 41, 43, 45, 53, 54, 57, 58

Definition A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

Recall



$D = (-\infty, 7)$

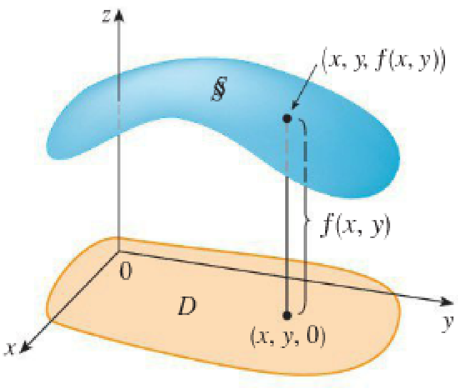


FIGURE 5

$z = 4y^2 + K$

$z = x^2 + K$

$z = x^2 + 4y^2$

Definition If f is a function of two variables with domain D, then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

$(x, y, f(x, y))$

EXAMPLE 5 Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$.

$z = 6 - 3x - 2y$

$3x + 2y + z = 6$

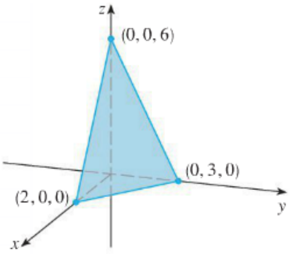
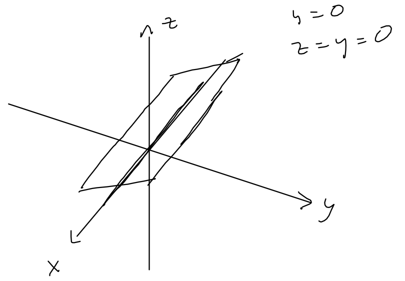


FIGURE 6

$f(x, y) = y$ is what?

$y = 0$

$z = y = 0$



EXAMPLE 6 Sketch the graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

$\mathcal{D}$ = Domain: Need $9 - x^2 - y^2 \geq 0$

$9 - x^2 - y^2 \geq 0$

$x^2 + y^2 \leq 9$

INSIDE!

Boundary:

$-x^2 - y^2 + 9 = 0$

$x^2 + y^2 = 9$

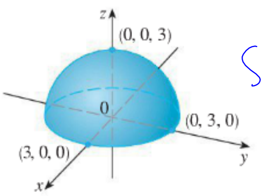
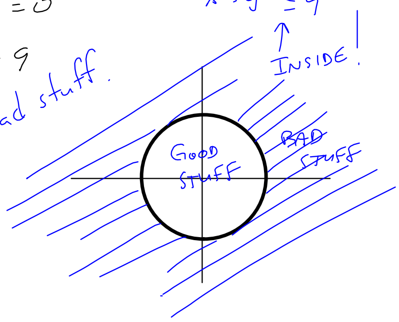


FIGURE 7
Graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$

Scratch out the bad stuff.



$z = 0$:

$\sqrt{9 - x^2 - y^2} = 0$

$x^2 + y^2 = 9$

$\sqrt{\ }$ = principle square root ≥ 0, always (when real)

$z = \pm 1$ No!

$\sqrt{9 - x^2 - y^2} = \pm 1$

$x^2 + y^2 + (\pm 1)^2 = 9$

$z \geq 0$

$x^2 = 9$

$\sqrt{x^2} = \sqrt{9}$

$|x| = \sqrt{9}$

$x = \pm\sqrt{9}$

$x = \pm 3$

$\sqrt{3^2} = 3$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

$f(3) = 3$

$f(-3) = 3$

$z = 1$:

$\sqrt{9 - x^2 - y^2} = 1$

$9 - x^2 - y^2 = 1^2$

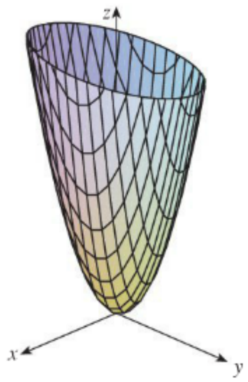
$-x^2 - y^2 = -8$

$x^2 + y^2 = 8$

$z = 2$ $\quad x^2 + y^2 = 5$

$z = 3$ $\quad x^2 + y^2 = 0$

EXAMPLE 8 Find the domain and range and sketch the graph of $h(x, y) = 4x^2 + y^2$.



$z = 4x^2 + y^2$ $\qquad$ $z = 0$ a point $(0,0,0)$

$z=1$: $4x^2+y^2 = 1$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2} + y^2 = 1$$

$z=2$: $\dfrac{x^2}{\left(\frac{1}{2}\right)^2} + y^2 = 2$ Bigger ellipse

$x = K$, $y = C$ for cross-sections.

Definition The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f).

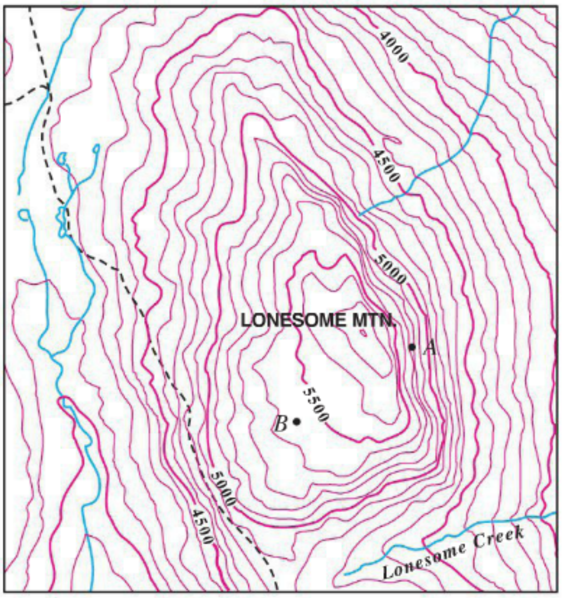


FIGURE 12

Topographic Maps!

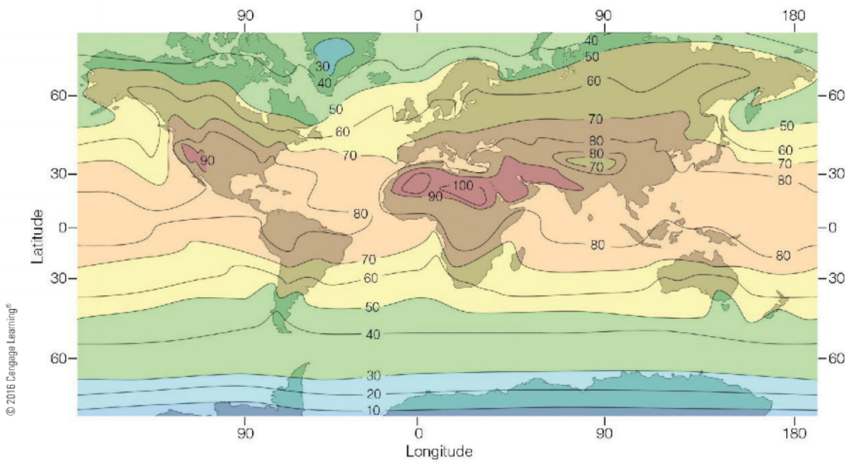


FIGURE 13 Average air temperature near sea level in July (°F)

EXAMPLE 12 Sketch some level curves of the function $h(x, y) = 4x^2 + y^2 + 1$

This is our friend, the elliptical paraboloid, opening up, with vertex at (0, 0, 1).

z = constant slices ∥ to xy-plane.

$$4x^2+y^2+1 = z$$

$$\frac{x^2}{\frac{1}{4}} + y^2 = z-1$$

$$\frac{x^2}{2^2}+\frac{y^2}{6^2} = 1$$

$$\frac{x^2}{\left(\frac{1}{2}\right)^2} + y^2 = z-1$$

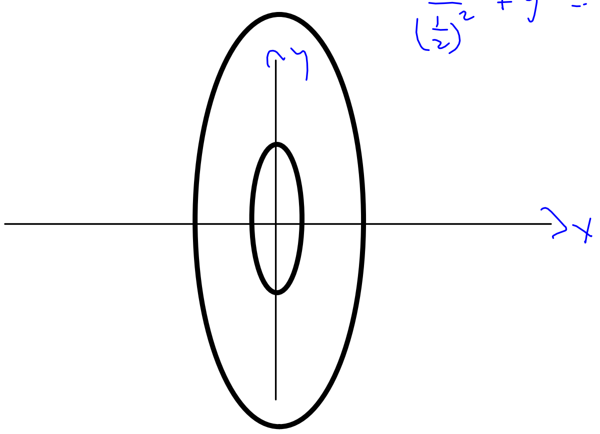
$z=0$: Nope

$z=1$ $(0,0,1)$

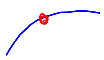
$z=2$: $\frac{x^2}{\left(\frac{1}{2}\right)^2} + y^2 = 1$

$z=4$: $\frac{x^2}{\left(\frac{1}{2}\right)^2} + y^2 = 3$

$$\frac{x^2}{\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{y^2}{\left(\sqrt{3}\right)^2}$$



We parametrize
wrt s for curvature
b/c $\bar{f}(t)$ might be
faster than $\bar{g}(t)$



$$L = s = \int ds$$

$$\kappa = \frac{\|\bar{r}' \times \bar{r}''\|}{\|\bar{r}'\|^3} = \frac{d\bar{T}}{ds} \qquad s = \int_0^t \underbrace{\sqrt{f'^2 + g'^2 + g'^2}}_{\|\bar{r}'\|}\, du$$

$$\frac{ds}{dt} = \|\bar{r}'(t)\|$$