1–4 Evaluate the line integral by two methods: (a) directly and (b) using Green's Theorem.

3. $\oint_C xy\,dx + x^2y^3\,dy$,
C is the triangle with vertices (0, 0), (1, 0), and (1, 2)

5–10 Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

5. $\int_C xy^2\,dx + 2x^2y\,dy$,
C is the triangle with vertices (0, 0), (2, 2), and (2, 4)

7. $\int_C \left(y + e^{\sqrt{x}}\right)dx + (2x + \cos y^2)\,dy$,
C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$

8. $\int_C xe^{-2x}\,dx + (x^4 + 2x^2y^2)\,dy$,
C is the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$

11–14 Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Check the orientation of the curve before applying the theorem.)

12. $\mathbf{F}(x, y) = \langle y^2 \cos x, x^2 + 2y \sin x \rangle$,
C is the triangle from (0, 0) to (2, 6) to (2, 0) to (0, 0)

14. $\mathbf{F}(x, y) = \langle y - \ln(x^2 + y^2), 2\tan^{-1}(y/x) \rangle$, C is the circle $(x - 2)^2 + (y - 3)^2 = 1$ oriented counterclockwise

27. If $\mathbf{F}$ is the vector field of Example 5, show that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every simple closed path that does not pass through or enclose the origin.

EXAMPLE 5 If $\mathbf{F}(x, y) = (-y\,\mathbf{i} + x\,\mathbf{j})/(x^2 + y^2)$, show that $\int_C \mathbf{F} \cdot d\mathbf{r} = 2\pi$ for every positively oriented simple closed path that encloses the origin.

Remember this if you're ever in Complex Analysis.