201 E4 C5

① $\int_0^3 (x^2+3x-5)\,dx$ $\Delta x = \frac{3-0}{n} = \frac{3}{n}$

$x_k = 0 + \frac{3k}{n} = \frac{3k}{n}$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n}\left(\left(\frac{3k}{n}\right)^2 + 3\left(\frac{3k}{n}\right) - 5\right)\left(\frac{3}{n}\right)$$

$$= \frac{3}{n}\left[\sum_{k=1}^{n}\left(\frac{9k^2}{n^2} + \frac{9k}{n} - 5\right)\right] = \frac{3}{n}\left[\frac{9}{n^2}\sum k^2 + \frac{9}{n}\sum k - \sum 5\right]$$

10pts

$$= \frac{3}{n}\cdot\frac{9}{n^2}\cdot\frac{n^3 + \text{smaller}}{3} + \frac{3}{n}\cdot\frac{9}{n}\cdot\frac{n^2+\text{smaller}}{2} - \frac{3}{n}\cdot 5n$$

$$\xrightarrow{n\to\infty} 9 + \frac{27}{2} - 15 = \frac{18+27-30}{2} = \frac{15}{2} = 7.5$$

$$\int_0^3 (x^2+3x-5)\,dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} - 5x\right]_0^3 = \frac{27}{3} + \frac{27}{2} - 15$$

10pts

$$= 9 + \frac{27}{2} - 15 = 7.5 \text{ OR } \frac{15}{2}$$

④ ⓐ $$\int_{-\sqrt{7}}^{0} t(t^2+1)^{\frac{1}{3}}\,dt = \frac{1}{2}\int (t^2+1)^{\frac{1}{3}}(2t\,dt) = \frac{1}{2}\left[\frac{3}{4}(t^2+1)^{4/3}\right]_{-\sqrt{7}}^{0}$$

15pts

$$= \frac{1}{2}\left[\frac{3}{4}(0^2+1)^{4/3} - \frac{3}{4}(8)^{4/3}\right] = \frac{1}{2}\cdot\frac{3}{4}\left[1-16\right]$$

$u = t^2+1$
$du = 2t\,dt$

$$= -\frac{3(15)}{8} = -\frac{45}{8} = -5.625$$

15pts

ⓑ $\int_{-\sqrt{7}}^{\sqrt{7}} t(t^2+1)^{\frac{1}{3}}\,dt = 0$. It's odd

ⓒ $$\int_0^{2\pi} \frac{\cos x}{\sqrt{3\sin x + 4}}\,dx = \frac{2}{3}\sqrt{3\sin x+4}\,\Big]_0^{2\pi} = \frac{2\sqrt{4}}{3} - \frac{2\sqrt{4}}{3} = 0$$

$u = 3\sin x + 4$
$du = 3\cos x\,dx$

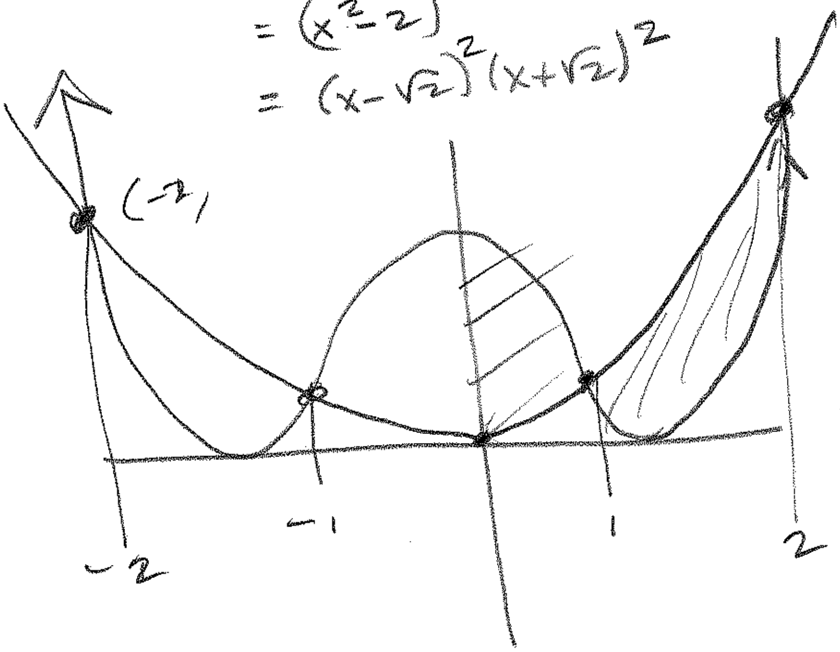
15pts

2.01 E4 Ⓒ5

(5) $y = x^4 - 4x^2 + 4$ & $y = x^2$

$= (x^2-2)^2$

$= (x-\sqrt{2})^2(x+\sqrt{2})^2$



Symmetry ⟹ Double it, side

$\frac{1}{2}\text{Area} = \int_0^1 f-g + \int_1^2 g-f$

$= \int_0^1 f-g - \int_1^2 f-g$

$x^4 - 4x^2 + 4 = x^2$ → $f(x) - g(x)$

$x^4 - 5x^2 + 4 = 0$

$(x^2-4)(x^2-1) = 0$

$(x-2)(x+2)(x-1)(x+1) = 0$

$\text{Area} = 2\int_0^1 (x^4 - 5x^2 + 4)\,dx - 2\int_1^2 (x^4 - 5x^2 + 4)\,dx$

$= 2\left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_0^1 - 2\left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_1^2$

$= 2\left[\frac{1}{5} - \frac{5}{3} + 4\right] - 2\left[\frac{32}{5} - \frac{40}{3} + 8 - \left(\frac{1}{5} - \frac{5}{3} + 4\right)\right]$

$= 4\left[\frac{3 - 25 + 60}{15}\right] - 2\left[\frac{96 - 200 + 120}{15}\right]$

$= 4\left[\frac{38}{15}\right] - 2\left[\frac{16}{15}\right] = \frac{152 - 32}{15} = \frac{120}{15} = \boxed{8}$

338 $\frac{4}{152}$

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10 pts

(2) $f(x) = x^2 + 3x - 5 \longrightarrow$

f_{AVG} on $[0,5]$ is

$$\frac{1}{5}\int_0^5 (x^2+3x-5)\,dx = \frac{1}{5}\left[\frac{x^3}{3} + \frac{3x^2}{2} - 5x\right]_0^5$$

$$= \frac{1}{5}\left[\frac{125}{3} + \frac{75}{2} - 25\right] = \left[\frac{25}{3} + \frac{15}{2} - 5\right]$$

$$= \frac{50+45-30}{6} = \boxed{\frac{65}{6}} \qquad \frac{54.1\overline{6}}{5} = 10.8\overline{33}$$

10 pts

(3) Set $x^2 + 3x - 5 = \frac{95}{6}$

$$x^2 + 3x = \frac{95+30}{6}$$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = \frac{125}{6} + \frac{9}{4} = \frac{250+27}{12} = \frac{277}{12}$$

$$\left(x + \frac{3}{2}\right)^2 = \frac{277}{12}$$

$$x + \frac{3}{2} = \pm\sqrt{\frac{277}{12}} = \frac{-3}{2} \pm \frac{\sqrt{277}}{2\sqrt{3}} \approx \begin{cases} -6.304511 77 \\ +3.304511768 \end{cases}$$

$$x = -\frac{3}{2} \pm \sqrt{\frac{277}{12}} \quad \text{OR} \quad -\frac{3}{2} \pm \frac{\sqrt{277}}{2\sqrt{3}}$$

$$\text{OR} \quad -\frac{3}{2} \pm \frac{\sqrt{831}}{2\cdot 3} = \frac{-9 \pm \sqrt{831}}{6}$$ to look "nice"

We want the positive one.