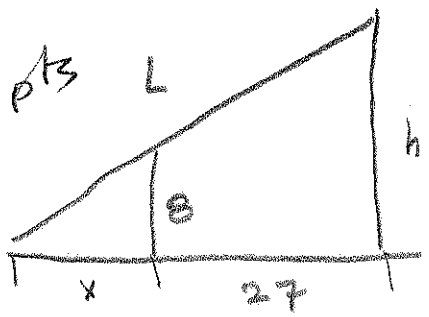
-01 TEST 3, Ch 4

30 pts



$L = \sqrt{h^2 + (x+27)^2}$

$L^2 = h^2 + (x+27)^2$ to be minimized.

Similar triangles:

$\frac{h}{x+27} = \frac{8}{x} \Rightarrow h = 8\left(\frac{x+27}{x}\right) = 8\left(1+\frac{27}{x}\right)$

$\Rightarrow f(x) = L^2(x) = \left(8\left(1+\frac{27}{x}\right)\right)^2 + (x+27)^2 \Rightarrow$

$f'(x) = 64\left(2\left(1+\frac{27}{x}\right)\left(-\frac{27}{x^2}\right)\right) + 2(x+27) \overset{SET}{=} 0$

$\Rightarrow 64\left(\frac{x+27}{x}\right)\left(-\frac{27}{x^2}\right) + (x+27) = 0$

$\Rightarrow (x+27)\left[\left(\frac{64}{x}\right)\left(-\frac{27}{x^2}\right) + 1\right] = 0$

$\Rightarrow (x+27)\left(\frac{-1728}{x^3} + \frac{x^3}{x^3}\right) = 0$

$\Rightarrow x = -27$ ~~OR~~ $x^3 = 1728$

$x = \sqrt[3]{1728}$

$= \sqrt[3]{2^6 \cdot 3^3} = 2^2 \cdot 3^1 = 12$

$\Rightarrow h = 8\left(\frac{12+27}{12}\right) = \frac{2}{8}\left(\frac{39}{12}\right) = 26 \Rightarrow$

$L = \sqrt{26^2 + 39^2}$

$= \sqrt{(13\cdot 2)^2 + (13\cdot 3)^2}$

$= \sqrt{13^2 \cdot 2^2 + 13^2 \cdot 3^2}$

$= \sqrt{13^2(2^2+3^2)} = 13\sqrt{13}$

≈ 46.872166588

$27 = 3^3$

$64 = 2^6$

$108 = 2^2 \cdot 3^3$

$1620 = 2^2 \cdot 3^4 \cdot 5$

$1728 = 2^6 \cdot 3^3$

$864 = 2^5 \cdot 3^3$

$432 = 2^4 \cdot 3^3$

$216 = 2^3 \cdot 3^3$

$108 = 2^2 \cdot 3^3$

$54 = 2 \cdot 3^3$

$27 = 3^3$

$9 = 3^2$

3

201 E3 C4

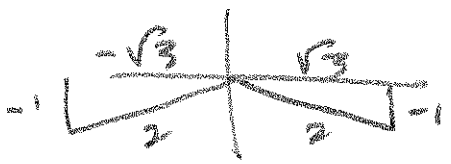
(2) $f(x) = \cos x - \frac{1}{2}x$ on $[0, 2\pi]$

$f(0) = 1 \rightsquigarrow (0, 1)$ $\quad f(2\pi) = 1 - 2\pi \rightsquigarrow (2\pi, 1-2\pi)$

$f'(x) = -\sin x - \frac{1}{2} \overset{\text{SET}}{=} 0$

$\sin x = -\frac{1}{2}$

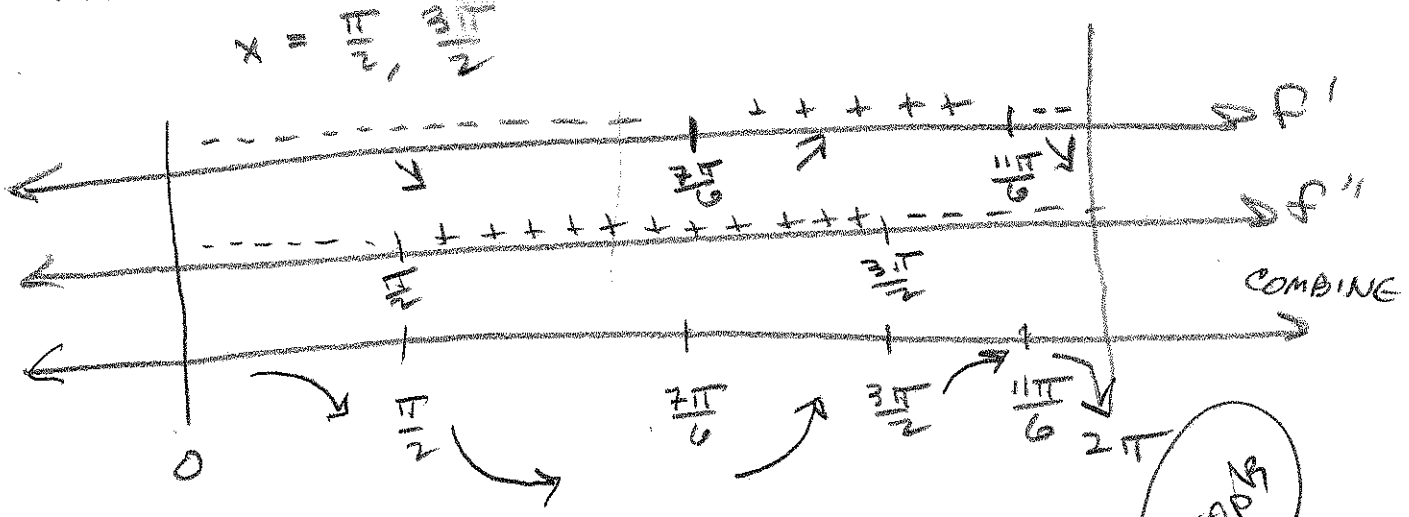
$\rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$



These are 30-60 triangles

$f''(x) = -\cos x \overset{\text{SET}}{=} 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$



$f(0) = 1 \rightarrow (0, 1) = A$

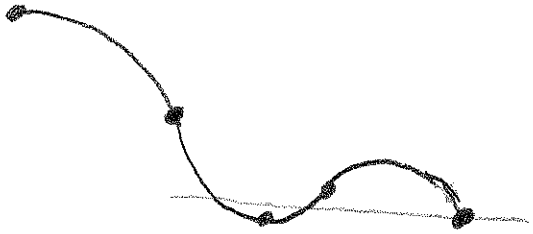
$f(\frac{\pi}{2}) = -\frac{\pi}{4} \rightarrow (\frac{\pi}{2}, -\frac{\pi}{4}) \approx (\frac{\pi}{2}, -.785398) = B$ IP

$f(\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2} - \frac{7\pi}{12} \rightarrow (\frac{7\pi}{6}, -\frac{\sqrt{3}}{2} - \frac{7\pi}{12}) \approx (\frac{7\pi}{6}, -2.698621) = C$ MIN

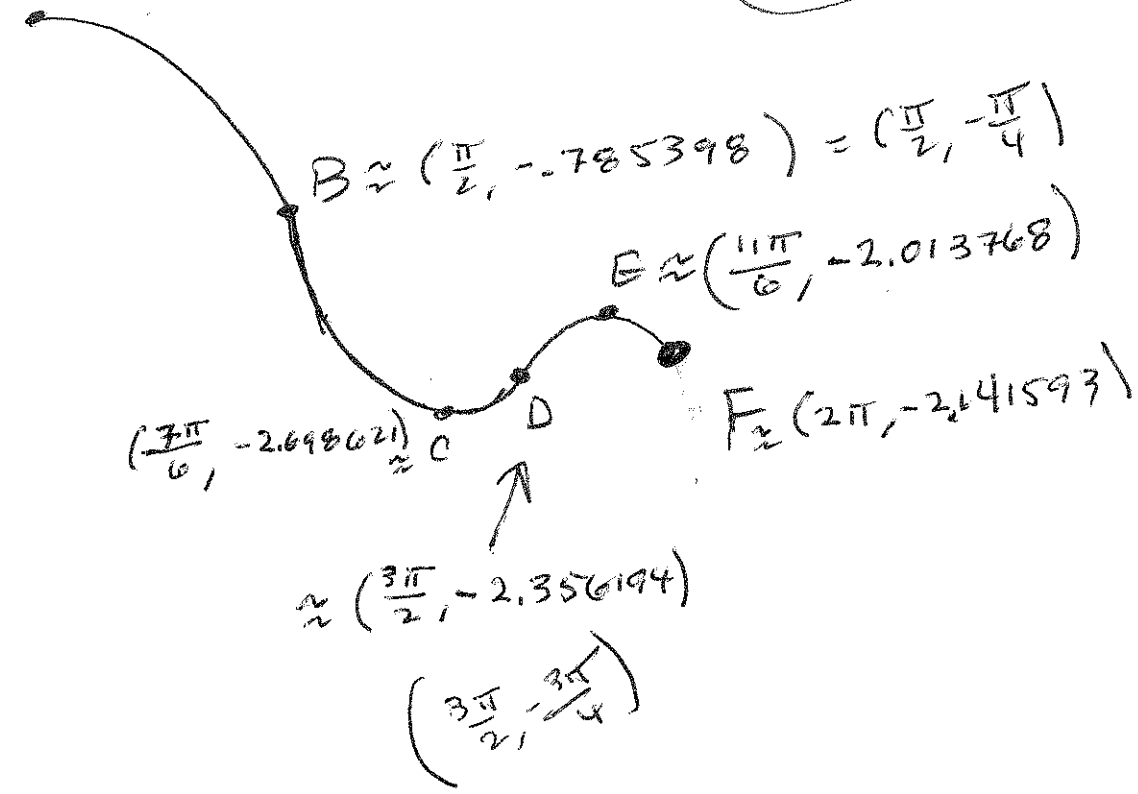
$f(\frac{3\pi}{2}) = -\frac{3\pi}{4} \rightarrow (\frac{3\pi}{2}, -\frac{3\pi}{4}) \approx (\frac{3\pi}{2}, -2.356194) = D$ IP

$f(\frac{11\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{11\pi}{12} \rightarrow (\frac{11\pi}{6}, \frac{\sqrt{3}}{2} - \frac{11\pi}{12}) \approx (\frac{11\pi}{6}, -2.013760)$
$= E$ MAX

$f(2\pi) = 1 - 2\pi \rightsquigarrow (2\pi, 1-2\pi)$
$\approx (2\pi, -2.141593)$



40 pts


A = (0,1)
B ≈ (π/2, -.785398) = (π/2, -π/4)
E ≈ (11π/6, -2.013768)
F ≈ (2π, -2.141593)
(7π/6, -2.698621) ≈ C
D
≈ (3π/2, -2.356194)
(3π/2, -3π/4)

201 E3 Q4

(3)

$f(-2) = 8$

$f(0) = 4$

$f(2) = 0$

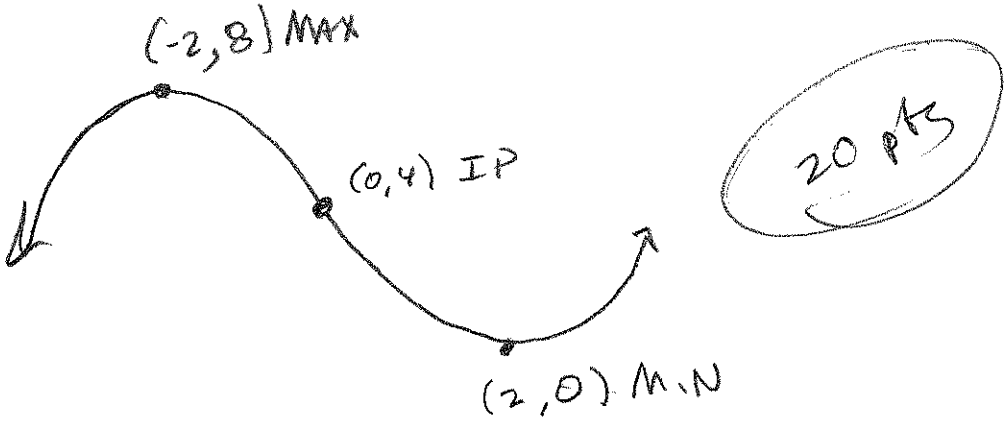
$f'(x) > 0 \quad \forall \; |x| > 2$

$f'(2) = f'(-2) = 0$

$f'(x) < 0 \quad \forall \; |x| < 2$

$f''(x) < 0 \quad \forall \; x < 0$

$f''(x) > 0 \quad \forall \; x > 0$



20 pts

201 E3 C4

(4) (a) $\frac{d}{dx}\left[\frac{3x^2+5x+2}{2x^2+x-1}\right]$

$= \frac{(6x+5)(2x^2+x-1) - (3x^2+5x+2)(4x+1)}{(2x^2+x-1)^2}$ 5pts

(b) $\frac{d}{dx}\left[(x^2-3x)\cos(x^2-3x)\right]$

$= (2x-3)\cos(x^2-3x) + (x^2-3x)(-\sin(x^2-3x))(2x-3)$ 5pts

(c) $\frac{d}{dx}\left[\sec^2(x^2-3x)\right]$

$= (2\sec(x^2-3x))(\sec(x^2-3x)\tan(x^2-3x))(2x-3)$ 5pts