201 TEST 2 PRAXIS

(1) $f(x) = 2x^3 - \frac{13}{2}x^2 - 28x + 5 \Longrightarrow$

$(3)(2)$

$f'(x) = 6x^2 - 13x - 28 \overset{\text{SET}}{=} 0 \Longrightarrow$ $(2)(2)(7)$

$6x^2 - 21x + 8x - 28 = 0$

$3x(2x-7) + 4(2x-7) = 0$

$(2x-7)(3x+4) = 0$

$x = \frac{7}{2}, -\frac{4}{3}$

$f(0) = 5 \to (0,5)$

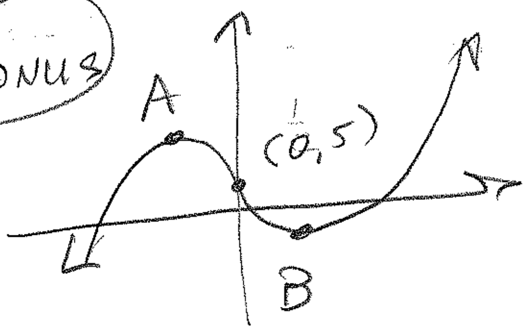
$f\left(\frac{7}{2}\right) = -\frac{695}{8} = -86.875$

$\boxed{\left(\frac{7}{2}, \frac{-695}{8}\right)} = B$

$f\left(-\frac{4}{3}\right) = \frac{703}{27} = 26.\overline{037}$

$\boxed{\left(-\frac{4}{3}, \frac{703}{27}\right)} = A$

BONUS



(2) (a) $V = \frac{4}{3}\pi r^3$

$$\left.\frac{dV}{dr}\right|_{r=3} = \left.4\pi r^2\right|_{r=3} = 4\pi(3)^2 = \boxed{36\pi}\ \frac{in^3}{in} = 36\pi\ in^2\ !?$$

(r is in inches)
(V is in inches^3)

(b) $$\left.\frac{dV}{dt}\right|_{r=3} = 20\ \frac{in^3}{s}\ \Big|_{r=3} = \left.4\pi r^2 \frac{dr}{dt}\right|_{r=3} = 36\pi \frac{dr}{dt} \longrightarrow$$

$$\left.\frac{dr}{dt}\right|_{r=3} = \frac{20}{36\pi} = \boxed{\frac{5}{9\pi}\ \frac{in}{s}} \approx .1768388257\ \frac{in}{s}$$

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(3) $f(x) = \dfrac{5}{\sqrt{2x-5}+2} = 5\left(\sqrt{2x-5}+2\right)^{-1}$

(a) $f'(x) = -5\left(\sqrt{2x-5}+2\right)^{-2}\left(\frac{1}{2}(2x-5)^{-\frac{1}{2}}(2)\right)$

$$= \frac{-5}{\left(\sqrt{2x-5}+2\right)^2\sqrt{2x-5}}$$

$$f'(7) = \frac{-5}{\left(\sqrt{9}+2\right)^2\sqrt{9}} = \frac{-5}{5^2 \cdot 3} = \frac{-1}{15} = m_{\tan}$$

$\Rightarrow$ @ $(7,1)$, we have tan line

$$\boxed{y = -\frac{1}{15}(x-7)+1}$$

(b) $f(17) = \dfrac{5}{\sqrt{2(17)-5}+2} = \dfrac{5}{\sqrt{29}+2} \approx .6770329614$

(c) $f'(17) = \dfrac{-5}{\left(\sqrt{29}+2\right)^2\left(\sqrt{29}\right)} \approx -.0170235693$

(d) $\left.\dfrac{df}{dx}\right|_{x=17} = f'(17) \approx -.0170235693$

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4 (a) $y = \dfrac{2x^2+x-1}{3x^2+5x+2} = \dfrac{(2x-1)(x+1)}{(3x+2)(x+1)}$

$= \dfrac{2x-1}{3x+2}$

$\rightarrow \boxed{y' = \dfrac{2(3x+2)-(2x-1)(3)}{(3x+2)^2}} = \dfrac{6x+4-6x+3}{(3x+2)^2}$

$= \dfrac{7}{(3x+2)^2}$

$\boxed{y' = \dfrac{(4x+1)(3x^2+5x+2)-(2x^2+x-1)(6x+5)}{3x^2+5x+2}}$

(b) $y = \dfrac{2x-1}{3x+2} \Longrightarrow y' = \dfrac{2(3x+2)-(2x-1)(3)}{(3x+2)^2} = \dfrac{7}{(3x+2)^2}$

(c) $y = \dfrac{5x+1}{2\sqrt{x}} = \dfrac{5}{2}x^{\frac{1}{2}} + \dfrac{1}{2}x^{-\frac{1}{2}}$

$\rightarrow \boxed{y' = \dfrac{5}{4}x^{-\frac{1}{2}} - \dfrac{1}{4}x^{-3/2}}$

OR $y' = \dfrac{5(2\sqrt{x})-(5x+1)(2(\frac{1}{2})x^{-\frac{1}{2}})}{4x}$

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(d) $y = \frac{2\sqrt{x}}{5x+1}$ $\longrightarrow$

$$\boxed{y' = \frac{2(\frac{1}{2})(x^{-1/2})(5x+1) - (2\sqrt{x})(5)}{(5x+1)^2}}$$

$$= \frac{\frac{5x+1}{\sqrt{x}} - 10\sqrt{x}}{(5x+1)^2} = \frac{\frac{5x+1-10x}{\sqrt{x}}}{(\quad)^2} = \frac{\frac{-5x+1}{\sqrt{x}}}{(5x+1)^2}$$

$$= \frac{-5x+1}{\sqrt{x}\,(5x+1)^2}$$

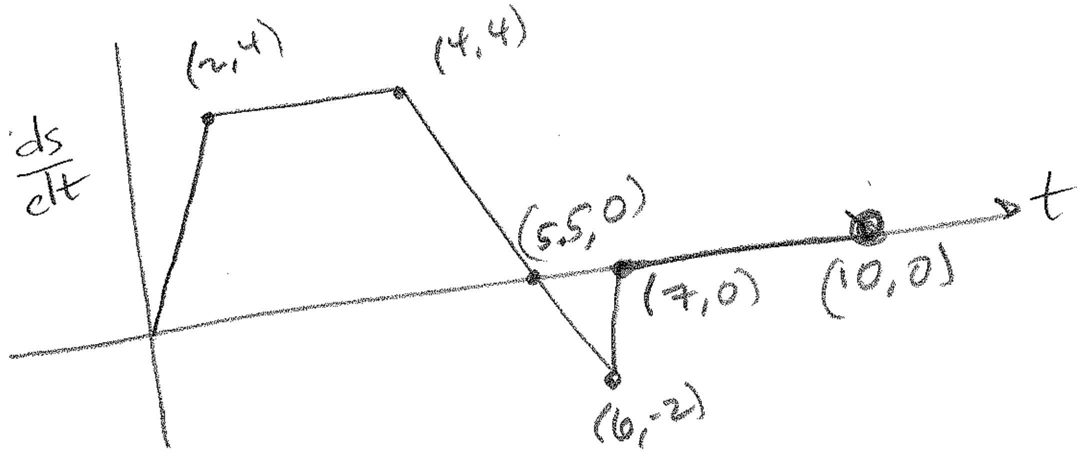
(e) $y=(x^2-3x)\sin(x^2-3x)$ $\longrightarrow$

$y'=(2x-3)\sin(x^2-3x) + (x^2-3x)(\cos(x^2-3x))(2x-3)$

(f) $y=\csc^2(x^2-3x)$ $\longrightarrow$

$y'=(2\csc(x^2-3x))(-\csc(x^2-3x)\cot(x^2-3x))(2x-3)$

(5)



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5 (a) Reverses Direction @ $t \approx 5.5s$

(b) Standing still for $t \in [7, 10]$

(d) For $t \in [4.5]$, it's slowing down.

(c) Velocity is constant for $t \in [2,4] \cup [7,10]$

e) @ $t = 6$ seconds, it starts slowing down. still moving "backwards", but slowing.

$\boxed{\text{BONUS:}}$ Net distance?

$$\frac{1}{2}(2)(4) + (2)(4) + \frac{1}{2}(\overset{\approx 3}{1.5})(4) - \frac{1}{2}(1.5)(2) = 14.5$$

Area under the curve.

$\boxed{\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \in \text{graph}}$

6 (a) $x\sin(2y) = y\cos(2x)$

$$\frac{\pi}{4}\sin\left(2 \cdot \frac{\pi}{2}\right) = \frac{\pi}{2}\cos\left(2\left(\frac{\pi}{4}\right)\right)$$

$$0 = 0 \checkmark$$

(b) $1\sin(2y) + x(2\cos(2y))y' = y'\cos(2x) - y(2\sin(2x))$

$$2x\cos(2y)y' - \cos(2x)y' = -\sin(2y) - 2\sin(2x)$$

$$\boxed{y' = \frac{-\sin(2y) - 2\sin(2x)}{2x\cos(2y) - \cos(2x)}}$$

(6) (c) $y'\Big|_{(\frac{\pi}{4},\frac{\pi}{2})} = \dfrac{-\sin(2(\frac{\pi}{2})) - 2\sin(2(\frac{\pi}{4}))}{2(\frac{\pi}{4})\cos(2(\frac{\pi}{2})) - \cos(2(\frac{\pi}{4}))}$

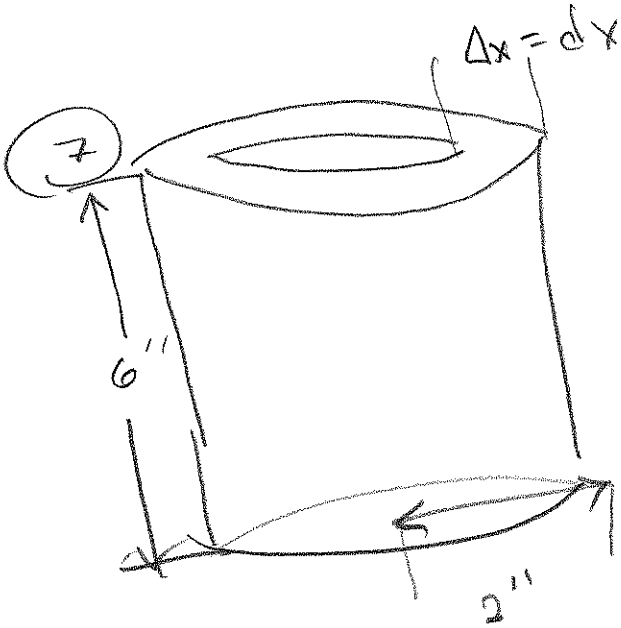
$= \dfrac{-0-2\cdot 1}{\frac{\pi}{2}(-1)-0} = \dfrac{-2}{-\frac{\pi}{2}} = \dfrac{4}{\pi} = m_{tan}$

$\boxed{y = \frac{4}{\pi}\left(x-\frac{\pi}{4}\right) + \frac{\pi}{2}}$ is tan. line

$= \frac{4}{\pi}x - 1 + \frac{\pi}{2}$

(d) Normal line is $\boxed{y = -\frac{\pi}{4}\left(x-\frac{\pi}{4}\right) + \frac{\pi}{2}}$

(7)



want $dV \approx \Delta V \le .05\,V$

$V = \pi r^2 h$

$V = 6\pi r^2$

$dV = 12\pi r\,dr \approx \Delta V \le .05\,V$

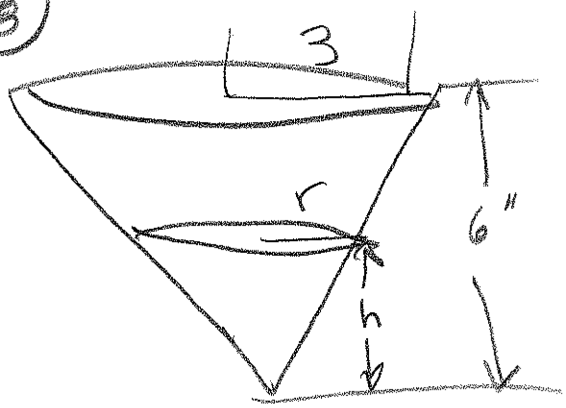
$\Longrightarrow 12\pi(2)\,dr \le .05\left(\pi(2)^2(6)\right)$

$dr \le \dfrac{(.05)\cancel{(6)}\cancel{\pi}(\cancel{2})^2}{\cancel{12}(2)\cancel{\pi}}$

$= .05$ in is approx. allowable error.

201 Test 2 Praxis

(8)



$\frac{dV}{dt} = 10 \frac{in^3}{min}$

Find $\frac{dh}{dt}$ when $h = 5$ in

(a) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{1}{12}\pi h^3 \longrightarrow$

$\frac{dV}{dt} = 10 = \frac{1}{4}\pi h^2 \frac{dh}{dt} = 10 \longrightarrow$

$\left(\frac{1}{4}\right)\pi(5)^2 \frac{dh}{dt}\Big|_{h=5} = 10 \longrightarrow$

$\frac{dh}{dt}\Big|_{h=5} = \boxed{\frac{40}{25\pi} \text{ in/min}} \approx .5092958179$ in/min.

(b) Find how fast h is rising in cylindrical coffeepot if $\frac{dV}{dt} = 10$ in³/min and pot is 6" in diameter?

r = 3 is constant, so...

$V = \pi r^2 h \Longrightarrow$

$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$, $\frac{dr}{dt} \to 0$

$\pi r^2 \frac{dh}{dt} = 9\pi \frac{dh}{dt} = 10 \longrightarrow \frac{dh}{dt} = \frac{10}{9\pi}$ in/min

$\approx .3536776513$ in/min

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9 $\sin(62^\circ) = ?$

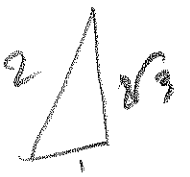
$f(x) = \sin x$ $\qquad x_0 = 60^\circ = \frac{\pi}{3}$ rads

$f'(x) = \cos x$ $\qquad \Delta x = dx = (2^\circ)\left(\frac{\pi \text{ rads}}{180^\circ}\right)$

$= \frac{\pi}{90}$ rads

$f(x+\Delta x) \approx f(x) + f'(x)\,dx$

$= \sin\left(\frac{\pi}{3}\right) + \left(\cos\left(\frac{\pi}{3}\right)\right)\left(\frac{\pi}{90}\right)$



$= \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{\pi}{90}\right) = \boxed{\frac{\sqrt{3}}{2} + \frac{\pi}{180}} \approx .8834787$

Check: $\sin 62^\circ \approx .8829475929$

10 $L(x) = \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

$L\left(\frac{\pi}{3} + \frac{\pi}{90}\right) = \frac{1}{2}\left(\frac{\pi}{3} + \frac{\pi}{90} - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

$= \frac{1}{2}\left(\frac{\pi}{90}\right) + \frac{\sqrt{3}}{2}$ = same as #9

11 Same as #10

$f(60^\circ) = \frac{\sqrt{3}}{2} = f\left(\frac{\pi}{3}\right)$

$f(x) = \sin x$ $\qquad f'(60^\circ) = \frac{1}{2} = f'\left(\frac{\pi}{3}\right)$

BONUS $\quad f'(x) = \cos x$ $\qquad f''(60^\circ) = -\frac{\sqrt{3}}{2} = f''\left(\frac{\pi}{3}\right)$

$f''(x) = -\sin x$

$Q_{\frac{\pi}{3}}(x) = f\left(\frac{\pi}{3}\right) + f'\left(\frac{\pi}{3}\right)\left(x - \frac{\pi}{3}\right) + \frac{f''\left(\frac{\pi}{3}\right)}{2}\left(x - \frac{\pi}{3}\right)^2$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{\pi}{90}\right) + \frac{-\frac{\sqrt{3}}{2}}{2}\left(\frac{\pi}{90}\right)^2 \approx \boxed{.8829510835}$

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<u>BONUS</u> The 6^{th} degree Taylor polynomial I <u>WANTED</u> was the <u>Maclaurin</u> polynomial, which just means expand @ $a=0$, instead of $a=\frac{\pi}{3}$. $a=\frac{\pi}{3}$ is a lot more work. * sigh*

$$f(x)=\sin x \qquad f\left(\tfrac{\pi}{3}\right)=\tfrac{\sqrt{3}}{2}$$
$$f'(x)=\cos x \qquad f^{(1)}\left(\tfrac{\pi}{3}\right)=\tfrac{1}{2}$$
$$f^{(2)}(x)=-\sin x \qquad f^{(2)}\left(\tfrac{\pi}{3}\right)=-\tfrac{\sqrt{3}}{2}$$
$$f^{(3)}(x)=-\cos x \qquad f^{(3)}\left(\tfrac{\pi}{3}\right)=-\tfrac{1}{2}$$
$$f^{(4)}(x)=\sin x \qquad f^{(4)}\left(\tfrac{\pi}{3}\right)=\tfrac{\sqrt{3}}{2}$$
$$f^{(5)}(x)=\cos x \qquad f^{(5)}\left(\tfrac{\pi}{3}\right)=\tfrac{1}{2}$$
$$f^{(6)}(x)=-\sin x \qquad f^{(6)}\left(\tfrac{\pi}{3}\right)=-\tfrac{\sqrt{3}}{2}$$

$$P(x)=\frac{\sqrt{3}}{2}+\frac{1}{2}\left(x-\frac{\pi}{3}\right)-\frac{\frac{\sqrt{3}}{2}}{2}\left(x-\frac{\pi}{3}\right)^2-\frac{\frac{1}{2}\left(x-\frac{\pi}{3}\right)^3}{6}$$
$$+\frac{\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{3}\right)^4}{24}+\frac{\frac{1}{2}\left(x-\frac{\pi}{3}\right)^5}{120}-\frac{\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{3}\right)^6}{720}$$

$$P\left(\frac{\pi}{3}+\frac{\pi}{90}\right)=P(62^\circ)\approx$$
$$\frac{\sqrt{3}}{2}+\frac{1}{2}\left(\frac{\pi}{90}\right)-\frac{\sqrt{3}}{4}\left(\frac{\pi}{90}\right)^2-\frac{1}{12}\left(\frac{\pi}{90}\right)^3+\frac{\sqrt{3}}{48}\left(\frac{\pi}{90}\right)^4$$
$$+\frac{1}{240}\left(\frac{\pi}{90}\right)^5-\frac{\sqrt{3}}{1440}\left(\frac{\pi}{90}\right)^6$$

ouch! $\approx .8829492\overset{0}{8}83$

Bonus cnt'd

The Maclaurin Polynomial, $a=0$ is

$f(x) = \sin x$ $\quad f(0)=0$

$f'(x) = \cos x$ $\quad f'(0)=1$

$f''(x) = -\sin x$ $\quad f''(0)=0$

$f^{(3)}(x) = -\cos x$ $\quad f'''(0)=-1$

$f^{(4)}(x) = \sin x$ $\quad 0$

$f^{(5)}(x) = \cos x$ $\quad 1 = f^{(5)}(0)$

$f^{(6)}(x) = -\sin x$ $\quad 0$

$$P(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$= x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

$$P\left(\frac{\pi}{3}+\frac{\pi}{90}\right) = P\left(\frac{30+1}{90}\pi\right) = P\left(\frac{31\pi}{90}\right)$$

$$= \frac{31\pi}{90} - \frac{\left(\frac{31\pi}{90}\right)^3}{6} + \frac{\left(\frac{31\pi}{90}\right)^5}{120} \approx .8832867552$$

Next Page for Maple Clobberage

Taylor Polynomial of degree 6, expanded @ $a = \frac{\pi}{3}$:

$$T := x \rightarrow \frac{\mathrm{sqrt}(3)}{2} + \frac{1}{2}\cdot\left(x - \frac{\mathrm{Pi}}{3}\right) - \frac{\mathrm{sqrt}(3)}{4}\cdot\left(x - \frac{\mathrm{Pi}}{3}\right)^2 - \frac{1}{23}\cdot\left(x - \frac{\mathrm{Pi}}{3}\right)^3 + \frac{\mathrm{sqrt}(3)}{48}\cdot\left(x - \frac{\mathrm{Pi}}{3}\right)^4 + \frac{1}{240}\cdot\left(x - \frac{\mathrm{Pi}}{3}\right)^5 - \frac{\mathrm{sqrt}(3)}{1440}\cdot\left(x - \frac{\mathrm{Pi}}{3}\right)^6$$

$$x \rightarrow \frac{1}{2}\sqrt{3} + \frac{1}{2}x - \frac{1}{6}\pi - \frac{1}{4}\sqrt{3}\left(x - \frac{1}{3}\pi\right)^2 - \frac{1}{23}\left(x - \frac{1}{3}\pi\right)^3 + \frac{1}{48}\sqrt{3}\left(x - \frac{1}{3}\pi\right)^4 + \frac{1}{240}\left(x - \frac{1}{3}\pi\right)^5 - \frac{1}{1440}\sqrt{3}\left(x - \frac{1}{3}\pi\right)^6 \tag{1}$$

$$T\left(\frac{\mathrm{Pi}}{3} + \frac{\mathrm{Pi}}{90}\right)$$

$$\frac{1}{2}\sqrt{3} + \frac{1}{180}\pi - \frac{1}{32400}\sqrt{3}\,\pi^2 - \frac{1}{16767000}\pi^3 + \frac{1}{3149280000}\sqrt{3}\,\pi^4 + \frac{1}{1417176000000}\pi^5 - \frac{1}{765275040000000}\sqrt{3}\,\pi^6 \tag{2}$$

evalf (%)

$$0.8829492883 \tag{3}$$

Maclaurin Polynomial is same as Taylor Polynomial, only expanded @ $a = 0$.

$$M := x \rightarrow x - \frac{1}{6}\cdot x^3 + \frac{1}{120}\cdot x^5$$

$$x \rightarrow x - \frac{1}{6}x^3 + \frac{1}{120}x^5 \tag{4}$$

$$M\left(\frac{\mathrm{Pi}}{3} + \frac{\mathrm{Pi}}{90}\right)$$

$$\frac{31}{90}\pi - \frac{29791}{4374000}\pi^3 + \frac{28629151}{708588000000}\pi^5 \tag{5}$$

evalf (%)

$$0.8832867549 \tag{6}$$