MAT 201 Test 2
100 Points Name___________________________
Fall, 2012

Even though there are 3.1 questions on this test, you do *not* have to differentiate by the definition, *unless specifically asked to do so*.

1. At what *x*-values does $f(x) = 2x^3 - \frac{13}{2}x^2 - 28x + 5$ have horizontal tangents? Find the corresponding *y*-values and report the points of horizontal tangency as ordered pairs (points on the graph).

Bonus Use this information to give a rough sketch of $f(x)$. *y*-intercept is easy, but don't waste time on *x*-intercepts. At least not yet.

2. Inflating balloon questions (3.1 and 3.8 combined). A spherical balloon is being inflated.
   a. What is its rate of change in volume, with respect to the radius r, when $r = 3$ inches?

   b. Suppose the rate of change of volume is 20 cubic inches per second. How fast is the radius changing, when the radius is $r = 3$ inches?

3. Let $f(x) = \dfrac{5}{\sqrt{2x-5}+2}$
   a. Find an equation of the tangent line to $f(x)$ at the point (7, 1).
   b. What is $f(17)$?
   c. What is $f'(17)$?
   d. What is $\left.\dfrac{df}{dx}\right|_{x=17}$?

4. Find the first derivatives.

   a. $\dfrac{2x^2 + x - 1}{3x^2 + 5x + 2}$

   b. $\dfrac{2x-1}{3x+2}$

   c. $\dfrac{5x+1}{2\sqrt{x}}$

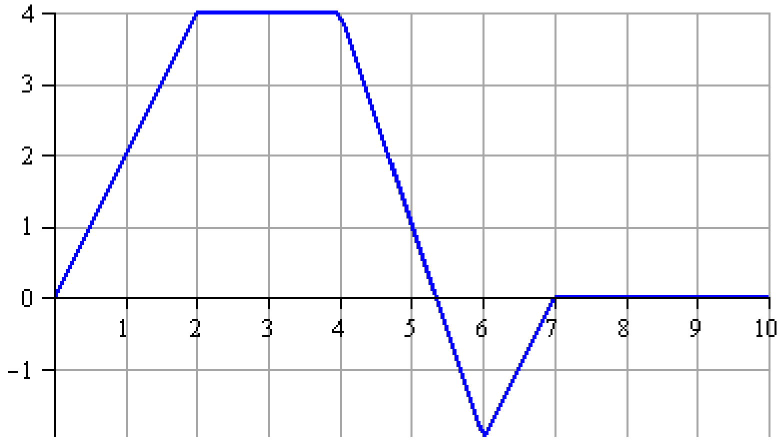
   d. $\dfrac{2\sqrt{x}}{5x+1}$

   e. $(x^2 - 3x)\sin(x^2 - 3x)$

   f. $\csc^2(x^2 - 3x)$

5. The figure, below shows the velocity $\dfrac{ds}{dt}$, in meters per second, of an object moving along a coordinate axis.
   a. When does it reverse direction?

b. When is it standing still?
c. During what time period(s) is its velocity constant?
d. What's happening between time $t = 4$ sec and $t = 5$ sec?
e. What happens at $t = 6$ seconds?



Bonus Can you tell the net distance traveled by the object? In other words, what is $s(10)$? What is its TOTAL distance, if you count back-tracking as positive distance? In other words, what is $|s(10)|$?

6. a. Verify that $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ is on the graph of the equation $x\sin(2y) = y\cos(2x)$.

b. Find $\frac{dy}{dx}$.

c. Find an equation of the tangent line to the curve at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

d. Find an equation of the normal line to the curve at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

7. Machinists are boring out an engine cylinder that's 6 inches deep. They need the radius of the cylinder to be 2 inches, and they need the volume of the cylinder to be within half a percent of the calculated volume that a 2-inch radius would produce. What is the allowable error in the radius of the cylinder? What is the maximum *percent* error in the radius of the cylinder?

8. A cone filter is 6 inches high and 6 inches in diameter. Coffee is draining from the cone filter into a cylindrical coffeepot that is *also* 6 inches in diameter. The coffee's dripping at a constant rate of 10 $\frac{in^3}{\min}$.

b. How fast is the level of coffee in the cone decreasing, when the level of coffee in the cone is 5 inches deep?

c. How fast is the level in the pot rising when the coffee in the cone is 5 inches deep? 2 inches deep? 3 inches deep?

9. Use differentials to approximate $\sin\left(62^0\right)$.
10. Use the linearization to approximate $\sin\left(62^0\right)$.
11. Use the tangent line to approximate $\sin\left(62^0\right)$

Bonus Use $a = 60^0$ and a *quadratic* approximation for $\sin\left(62^0\right)$.
Bonus Use the 6^{th} degree Taylor polynomial to approximate $\sin\left(62^0\right)$.

Other good stuff:

Differentials to estimate the volume of paint needed to cover an object with a coat of a desired thickness.
Differentials to approximate something like $\sqrt{103}$.