MAT 121 9:43 10:02, plus edit during Test 1
100 Points Name KEY
Fall, 2012

1. (5 pts each) Find the average rate of change of $f(x) = x^3 + 2x - 5$ over the intervals.

   a. [1, 1.1]

$f(1) = 1^3 + 2(1) - 5 = 3 - 5 = -2$

$f(1.1) = (1.1)^3 + 2(1.1) - 5 = -1.469$

$$\frac{f(1.1) - f(1)}{.1} = \frac{-1.469 + 2}{.1} = \boxed{5.31}$$

   b. [1, 1.001]

$f(1.001) = (1.001)^3 + 2(1.001) - 5 \approx -1.994996999$

$$\frac{f(1.001) - f(1)}{.001} \approx \frac{-1.994996999 + 2}{.001} \approx \boxed{5.003001}$$

2. (10 pts) Based on your work in #1 (and maybe a few more intervals), what would you estimate the rate of change of f is, at $x = 1$?

$\boxed{f'(1) = 5}$

3. (10 pts) Compute $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$ to find the slope of the curve of f at $x = 1$.

$$\frac{(1+h)^3 + 2(1+h) - 5 - (1^3 + 2(1) - 5)}{h}$$

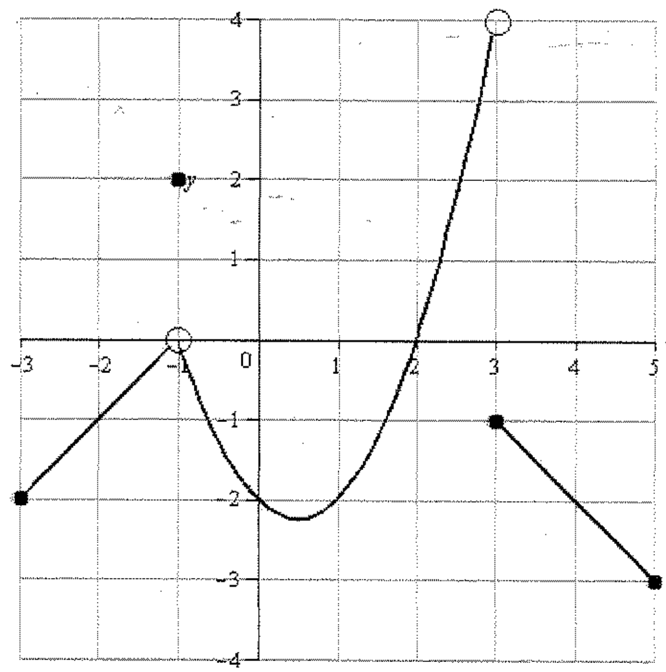
$$= \frac{1^3 + 3(1)^2(h) + 3(1)(h)^2 + h^3 + 2 + 2h - 5 - 1 - 2 + 5}{h}$$

$$= \frac{3h + 3h^2 + h^3 + 2h}{h} = 3 + 3h + h^2 + 2 \xrightarrow{h \to 0} 5$$

$\boxed{f'(1) = 5}$

4. (10 pts) Based on previous work, find the equation of the tangent line to f at $x = 1$.

5. (3 pts each) Use the graph of the function $f(x)$ to evaluate / answer the following:



a. $\lim\limits_{x \to 3^-} f(x)$ = 4

b. $\lim\limits_{x \to 3^+} f(x)$ = −1

c. $\lim\limits_{x \to -1} f(x)$ = 0

d. $\lim\limits_{x \to 0} f(x)$ = −2

e. Where is f continuous?

$[-3,-1) \cup (1,3) \cup (3,5]$

f. Where does f only have a left-hand limit?

@ $x=5$

g. Where does f have a removable discontinuity, and what would you define f to be at that point?

@ $x=-1$, define $f(-1)=0$.

6. (10 pts) Let $f(x) = x^2 - 2$. Find a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - x_0| < \delta$, for $x_0 = 3, L = 7$, and $\varepsilon = 0.3$.

want $|x^2 - 2 - 7| < .3$

$|x^2 - 9| < .3$

$-.3 < x^2 - 9 < .3$

$8.7 < x^2 < 9.3$

$\sqrt{8.7} < |x| = x < \sqrt{9.3}$

.05042376 .049590136

$\sqrt{8.7}$ $3 = x_0$ $\sqrt{9.3}$

Let $\boxed{\delta = .049590136}$

7. (10 pts) Prove that $\lim_{x\to 3}(5x-2)=13$.

Let $\varepsilon > 0$ be given.

Define $\delta = \frac{\varepsilon}{5}$.

Then $0 < |x-3| < \delta \Longrightarrow$

$$|5x-2-13| = |5x-15|$$
$$= 5|x-3|$$
$$< 5\delta$$
$$= \varepsilon \; \blacksquare$$

8. (5 pts each) Evaluate the limits:

a. $\lim_{x\to 2}\frac{x^2-5x+6}{x^2+5x-14} = \lim_{x\to 2}\frac{(x-3)(x-2)}{(x+7)(x-2)} = \lim_{x\to 2}\frac{x-3}{x+7}$

$$= \frac{2-3}{2+7} = \boxed{-\frac{1}{9}}$$

b. $\lim_{x\to 0}(\tan(3x)\cot(5x)) = \lim_{x\to 0}\frac{\sin(3x)}{\cos(3x)}\cdot\frac{\cos(5x)}{\sin(5x)}$

$$= \lim_{x\to 0}\frac{\sin(3x)}{x}\cdot\frac{x}{\sin(5x)}\cdot\frac{\cos(5x)}{\cos(3x)}\cdot\frac{3}{3}\cdot\frac{5}{5}$$

$$= \lim_{x\to 0}\frac{\sin(3x)}{3x}\cdot\frac{5x}{\sin(5x)}\cdot\frac{\cos(5x)}{\cos(3x)}\cdot\frac{3}{1}\cdot\frac{1}{5} = \boxed{\frac{3}{5}}$$

8. Cnt'd Evaluate the Limits

c. $\lim_{x\to\infty} \sqrt{9x^2 - x} - 3x$

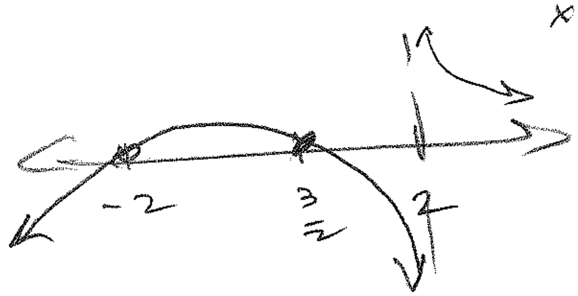
$$\left(\sqrt{9x^2-x}-3x\right)\left(\frac{\sqrt{9x^2-x}+3x}{\sqrt{9x^2-x}+3x}\right) = \frac{9x^2-x-9x^2}{\sqrt{9x^2-x}+3x} = \frac{x}{|x|\sqrt{9-\frac{1}{x^2}}+3x}$$

$|x| = x$ as $x \to$ big

$$= \frac{x}{x\left(\sqrt{9-\frac{1}{x^2}}+3\right)} = \frac{1}{\sqrt{9-\frac{1}{x^2}}+3} \xrightarrow{x\to\infty} \frac{1}{3+3} = \frac{1}{6}$$

9. (10 pts) Sketch the graph of $f(x) = \cancel{\frac{2x^2+x-6}{x+2}}$. Include all asymptotes and intercepts.

$$f(x) = \frac{2x^2+x-6}{x-2} = \frac{(2x-3)(x+2)}{x-2}$$



2	2	1	-6
		4	10
	2	5	4

$x = 2$

$\boxed{\text{O.A.: } y = 2x+5}$

$\overset{SET}{=} 0 \Rightarrow 2x = -5$

$x = -\frac{5}{2}$

$\boxed{\text{V.A.: } x = 2}$