

201 Midterm Re-do

1 a) $\frac{f(2.1) - f(2)}{2.1 - 2} = m_{AVG} = \frac{(2.1)^2 + 5(2.1) + 2 - (2^2 + 5(2) + 2)}{0.1} = 9.1$ 9.1 5pts

b) $\frac{f(2.001) - f(2)}{2.001 - 2} \approx 9.001$ 9.001 5pts

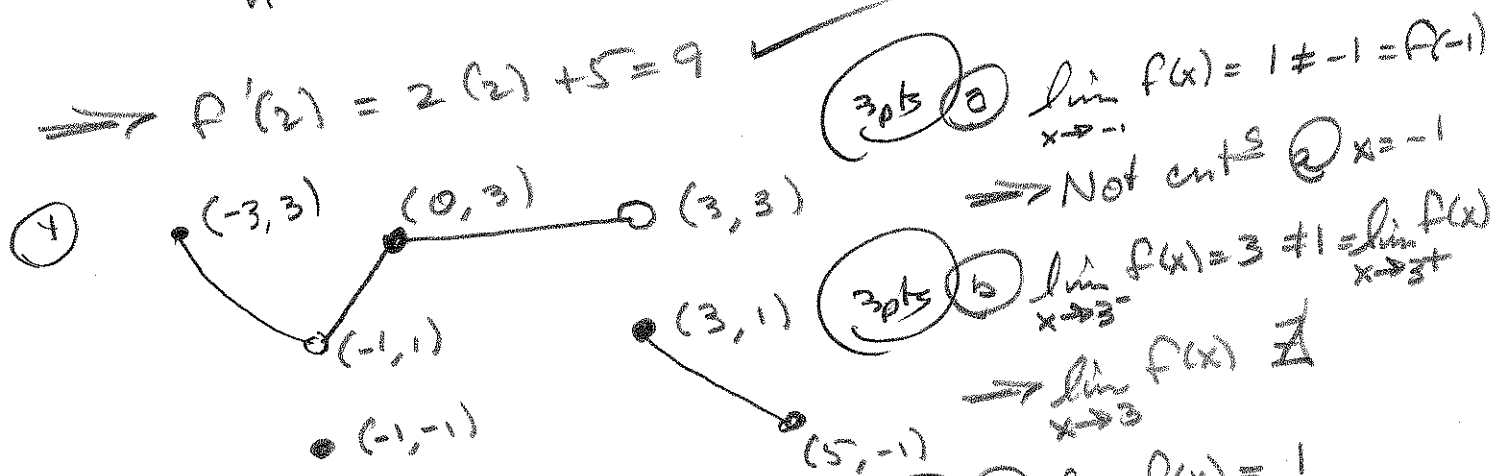
2 My guess is $f'(2) = 9$ 10pts

3 $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 5(x+h) + 2 - (x^2 + 5x + 2)}{h}$ 10pts

$= \frac{x^2 + 2xh + h^2 + 5x + 5h + 2 - x^2 - 5x - 2}{h} = \frac{2xh + h^2 + 5h}{h}$

$= \frac{h(2x + h + 5)}{h} \xrightarrow{h \rightarrow 0} 2x + 5 = f'(x)$

$\Rightarrow f'(2) = 2(2) + 5 = 9$ ✓



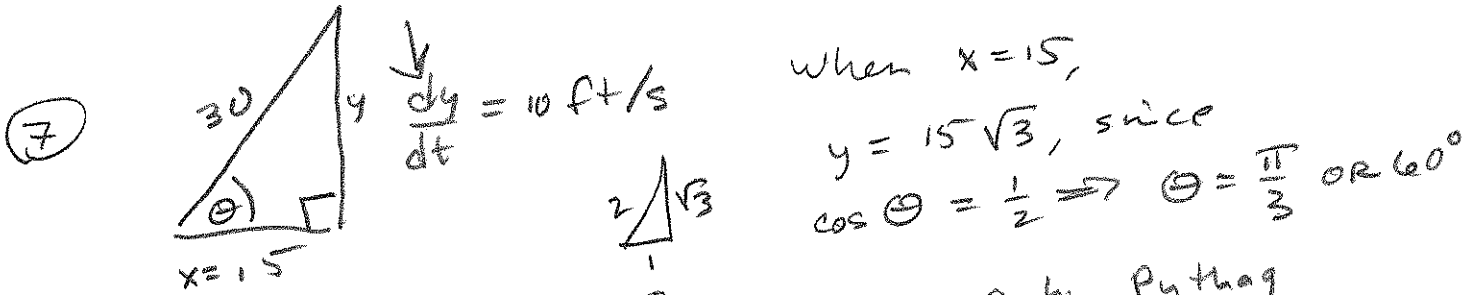
3pts c) Define $f(-1) = 1$ to make f continuous at $x = -1$.

3pts c) $\lim_{x \rightarrow -1} f(x) = 1$

5 $\lim_{x \rightarrow 3} (3x + 7) = 16$

PF Let $\epsilon > 0$ be given. Define $\delta = \frac{\epsilon}{3}$.
 Then if $0 < |x - 3| < \delta$, we have
 $|3x + 7 - 16| = |3x - 9| = 3|x - 3| < 3\delta = \epsilon$ QED

- (6) (a) $y = 3x^5 - 11x^2 + 4x^{-\frac{3}{2}} \Rightarrow y' = 15x^4 - 22x - 4(-\frac{3}{2})x^{-\frac{5}{2}}$ (6pts)
- (b) $y = \sin^3(3x^5 - 1) \rightarrow y' = (3\sin^2(3x^5 - 1))(\cos(3x^5 - 1))(15x^4)$ (6pts)
- (c) $y = \frac{\cos(3x)}{(x^2 - 2x)^{\frac{2}{3}}} \rightarrow y' = \frac{(-\sin(3x))(3)(x^2 - 2x)^{\frac{2}{3}} - (\cos(3x))(\frac{2}{3}(x^2 - 2x)^{-\frac{1}{3}})(2x - 2)}{(x^2 - 2x)^{\frac{4}{3}}}$ (6pts)



We know $x^2 + y^2 = 30^2$, so

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow$$

$$2x \frac{dx}{dt} = -2y \frac{dy}{dt} \Rightarrow$$

$$\frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt}$$

OR by Pythag
 $15^2 + y^2 = 30^2$
 $y^2 = 30^2 - 15^2 = 675$
 $y = 15\sqrt{3}$

$$\frac{dx}{dt} = -\frac{(15\sqrt{3})(10)}{15} = -10\sqrt{3} \text{ ft/s}$$

(8) $y = 4x^3 + 3x^2 - 3(4x - 27) = x^2(4x + 3) - 9(4x + 3)$
 $= (x - 3)(x + 3)(4x + 3)$ SET $= 0 \Rightarrow x = \pm 3, -\frac{3}{4}$

$$y' = 12x^2 + 6x - 36 \stackrel{\text{SET } = 0}{=} 0$$

$$\Rightarrow 2x^2 + x - 6 = 0$$

$$\Rightarrow (2x + 3)(x - 2) = 0$$

$$\Rightarrow x = -2, \frac{3}{2}$$

$$y'' = 24x + 6 \stackrel{\text{SET } = 0}{=} 0$$

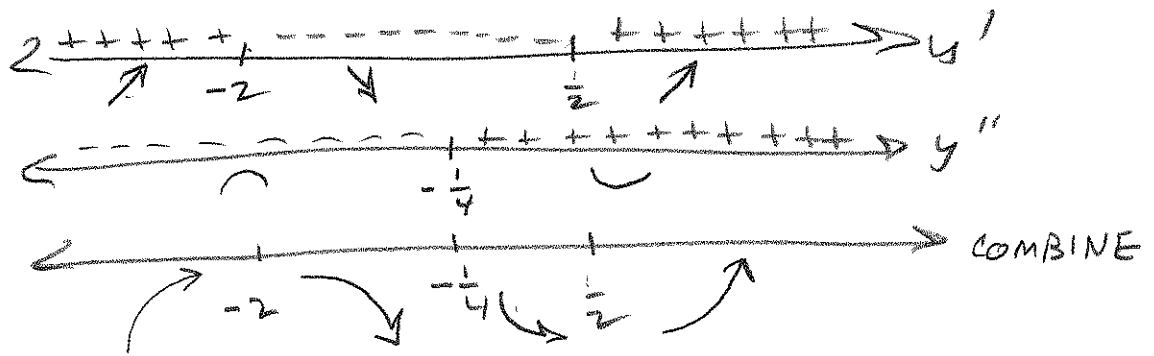
$$\Rightarrow 24x = -6$$

$$\Rightarrow x = -\frac{1}{4}$$

$$\begin{array}{r} 5 \overline{) 675} \\ \underline{5} \\ 175 \\ \underline{15} \\ 25 \\ \underline{24} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

COMBINE

8) cont'd



$f(-2)$:

-2	4	3	-36	-27
		-8	10	52
	4	-5	-26	$25 = f(-2)$

$\rightarrow (-2, 25) \text{ MAX}$

$f(\frac{3}{2})$:

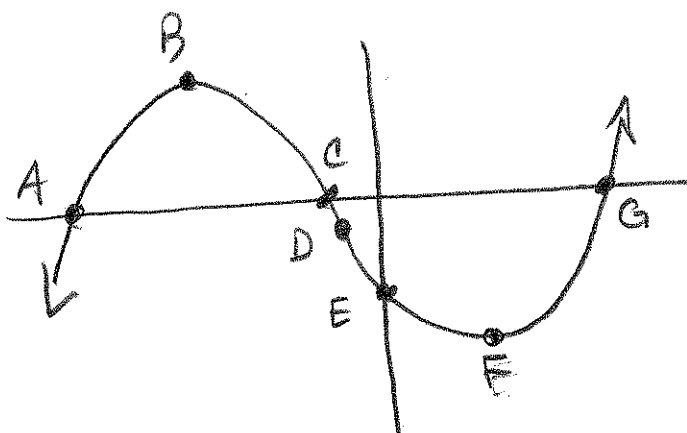
$\frac{3}{2}$	4	3	-36	-27
		6	$\frac{27}{2}$	$-\frac{135}{4}$
	4	9	$-\frac{45}{2}$	$-\frac{243}{4} = -60.75 = f(\frac{3}{2})$

$\rightarrow (\frac{3}{2}, -60.75) \text{ MIN}$

$f(-\frac{1}{4})$:

$-\frac{1}{4}$	4	3	-36	-27
		-1	$-\frac{1}{2}$	$\frac{73}{8}$
	4	2	$-\frac{73}{2}$	$-\frac{143}{8} = -17.875$

$\rightarrow (-\frac{1}{4}, -17.875) \text{ IP}$



- A = (-3, 0)
- B = (-2, 25) MAX
- C = (-3/4, 0)
- D = (-1/4, -17.875) IP
- E = (0, -27)
- F = (3/2, -60.75) MIN
- G = (3, 0)