201 FINAL FALL '12

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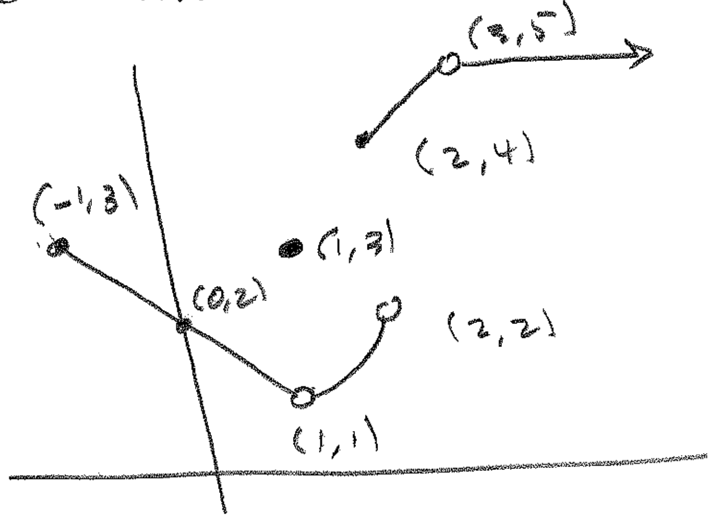

a) 5pts $\lim_{x\to 1} f(x) = 1$.

b) 5pts $f(1) = 3$

c) 5pts $\lim_{x\to 1} f(x) = 1 \neq 3 = f(1)$ is why

f isn't cnt's @ $x = 1$.

d) 5pts $\lim_{x\to 2^-} f(x) = 2 \neq 4 = \lim_{x\to 2^+} f(x)$ is why

$\lim_{x\to 2} f(x) \nexists$.

5pts e) Let $f(3) = 5$ to make f cnt's @ $x = 3$.

2) 10pts $\lim_{x\to 4} (5x+4) = 24$.

$\boxed{\text{Pf}}$ Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$. Then

$0 < |x-4| < \delta$ implies $|5x+4-24| = |5x-20|$

$= 5|x-4| < 5\delta = 5 \cdot \frac{\varepsilon}{5} = \varepsilon$ ∎

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3) 10 pts $f(x) = x^2 - 5x + 2 \implies$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) + 2 - (x^2 - 5x + 2)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h + 2 - x^2 + 5x - 2}{h}$$

$$= \frac{2xh + h^2 - 5h}{h} = \frac{h(2x + h - 5)}{h} = 2x + h - 5 \xrightarrow{h \to 0} \boxed{2x - 5}$$

$$= f'(x)$$

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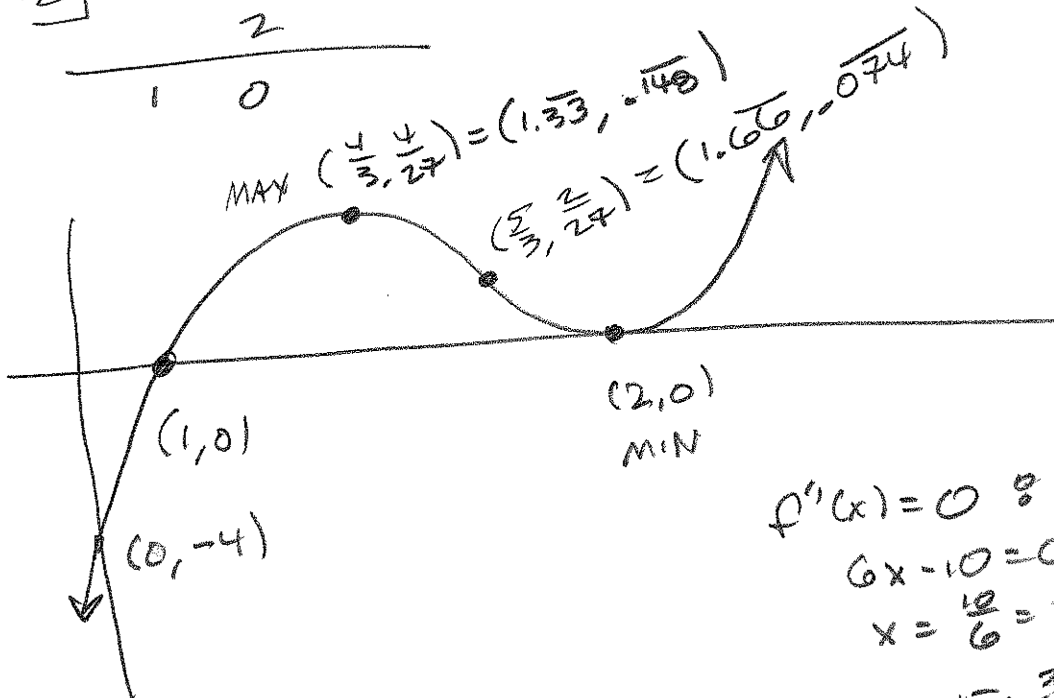
4. 15 pts $f(x) = x^3 - 5x^2 + 8x - 4$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 8 & -4 \\ & & 1 & -4 & 4 \\ \hline 2 & 1 & -4 & 4 & 0 \\ & & 2 & -4 & \\ \hline 2 & 1 & -2 & 0 & \\ & & 2 & & \\ \hline & 1 & 0 & & \end{array}$$

$x=1$ $\quad f(x) = (x-1)(x-2)^2$

$x=2$

$x=2$



$f''(x) = 0$:

$6x - 10 = 0 \longrightarrow$

$x = \frac{10}{6} = \frac{5}{3}$

$f'(x) = 3x^2 - 10x + 8$

$= (3x-4)(x-2) \overset{\text{SET}}{=} 0$

$x = \frac{4}{3}, 2$ are cps.

$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 5\left(\frac{4}{3}\right)^2 + 8\left(\frac{4}{3}\right) - 4$

$= \frac{64}{27} - 5\left(\frac{16}{9}\right) + \frac{32}{3} - 4$

$= \frac{64 - 240 + 288 - 108}{27} = \frac{4}{27} \doteq .\overline{148}$

$f\left(\frac{5}{3}\right) = \left(\frac{5}{3}\right)^3 - 5\left(\frac{5}{3}\right)^2 + 8\left(\frac{5}{3}\right) - 4 =$

$= \frac{125 - 375 + 360 - 108}{27}$

$= \frac{2}{27} \doteq .\overline{074}$

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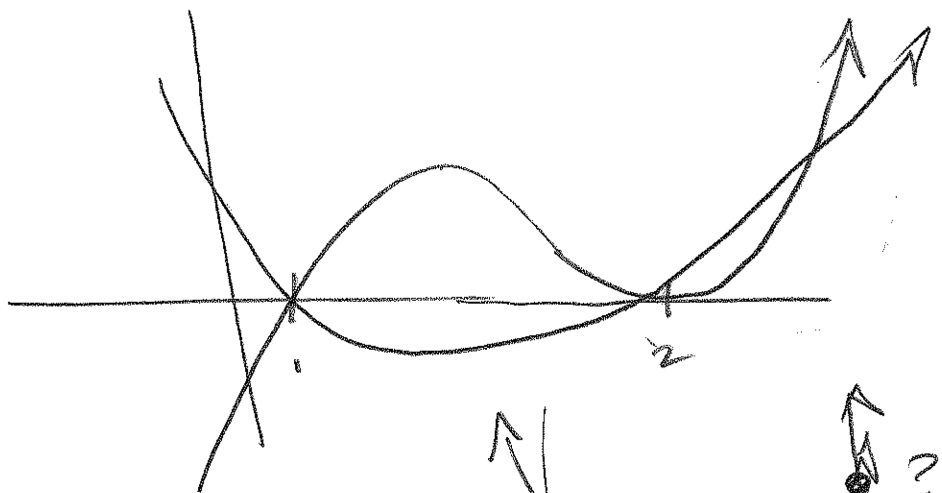
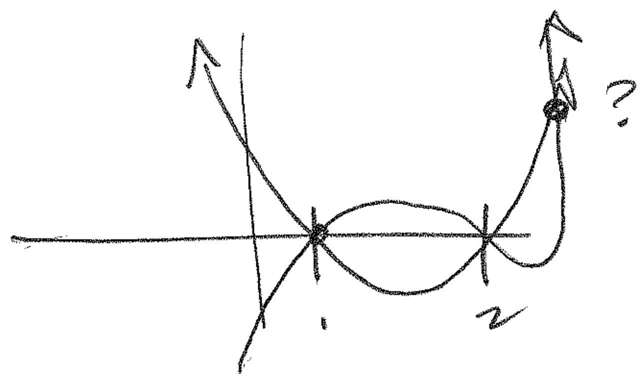
5 10 pts $f(x) = x^3 - 5x^2 + 8x - 4$

$g(x) = x^2 - 3x + 2 = (x-1)(x-2)$





Find intersection:

$x^3 - 5x^2 + 8x - 4 = x^2 - 3x + 2$

$x^3 - 6x^2 + 11x - 6 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline 2 & 1 & -5 & 6 & 0 \\ & & 2 & -6 & \\ \hline & 1 & -3 & 0 & \end{array}$$

$x - 3 = 0$

$x = 3$ is other crosser.

$$\int_1^2 (x^3 - 6x^2 + 11x - 6)\,dx - \int_2^3 (x^3 - 6x^2 + 11x - 6)\,dx$$

$$= \left[\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right]_1^2 - \left[\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right]_2^3$$

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5 cnt'd

$$= \left[\frac{2^4}{4} - 2\cdot(2)^3 + \frac{11}{2}(2)^2 - 6(2) - \left(\frac{1}{4} - 2 + \frac{11}{2} - 12\right)\right]$$

$$- \left[\frac{3^4}{4} - 2(3)^3 + \frac{11}{2}(3)^2 - 6(3) - \left(\frac{2^4}{4} - 2(2)^3 + \frac{11}{2}(2)^2 - 6(2)\right)\right]$$

$$= \left[\frac{16}{4} - 16 + \frac{44}{2} - 12 - \left(\frac{1}{4} + \frac{11}{2} - 14\right)\right]$$

$$- \left[\frac{81}{4} - 54 + \frac{99}{2} - 18 - \left(\frac{16}{4} - 16 + \frac{44}{2} - 12\right)\right]$$

$$= \frac{16+88}{4} - 28 - \left(\frac{1+22-56}{4}\right)$$

$$- \left[\frac{81+198}{4} - 34 - \left(\frac{16+88}{4} - 28\right)\right]$$

$$= \frac{104}{4} - 28 - \left(-\frac{33}{4}\right) - \left[\frac{279}{4} - 34 - \left(\frac{104}{4} - 28\right)\right]$$

$$= \frac{104}{4} + \frac{33}{4} - 28 - \frac{279}{4} + 34 + \frac{104}{4} - 28$$

$$= -\frac{38}{4} -$$

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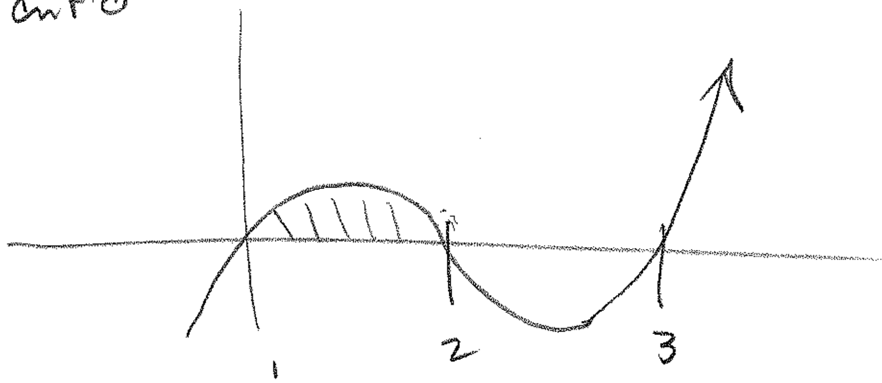
5 cont'd



$$\int_1^2 (x^3-6x^2+11x-6)\,dx - \int_2^3 (x^3-6x^2+11x-6)\,dx$$

$$= \left[\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right]_1^2 - \left[\left(\frac{1}{4}x^4 - 2x^3 + \frac{11}{2}x^2 - 6x\right)\right]_2^3$$

$$= \frac{16}{4} - 16 + \frac{11}{2}(4) - 6(2) - \left(\frac{1}{4} - 2 + \frac{11}{2} - 6\right)$$

$$- \left[\frac{1}{4}(81) - 2(27) + \frac{99}{2} - 18 - \left(\frac{16}{4} - 16 + \frac{11}{2}(4) - 6(2)\right)\right]$$

$$= 4-16+22-12 - \left(\frac{1-8+22-24}{4}\right)$$

$$- \left[\frac{81}{4} - 54 + \frac{198}{4} - 18 - (4-16+22-12)\right]$$

$$= 26-28-\left(-\frac{9}{4}\right) - \left[\frac{279}{4} - 72 - (26-28)\right]$$

$$= -2 + \frac{9}{4} - \frac{279}{4} + 70 = 68 - \frac{270}{4} = 68 - \frac{135}{2}$$

$$= \frac{136-135}{2} = \boxed{\frac{1}{2}}$$

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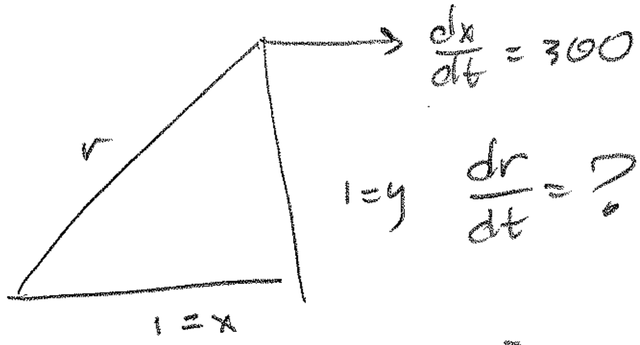

we know $x^2 + y^2 = r^2 \longrightarrow$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$$

$$2(1)(300) + 2(1)(0) = 2\sqrt{2}\,\frac{dr}{dt} \longrightarrow$$

$$\frac{dr}{dt} = \frac{600}{2\sqrt{2}} = \frac{300}{\sqrt{2}} = \frac{300\sqrt{2}}{2} = 150\sqrt{2} \approx 212.1320344 \text{ miles/hr}$$

7 $f(x) = \sqrt{x}$, $a = 36$, $\Delta x = 39 - 36 = 3 \longrightarrow$

$$f'(x) = \tfrac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}},$$

$$f(a) = \sqrt{36} = 6$$

$$f'(a) = \frac{1}{2\sqrt{36}} = \frac{1}{2(6)} = \frac{1}{12}$$

$$f(a + \Delta x) \approx f(a) + f'(a)\Delta x$$

$$= 6 + \left(\frac{1}{12}\right)(3) = 6 + \frac{3}{12} = 6 + \frac{1}{4} = \boxed{6.25}$$

$$L(x) = f(a) + f'(a)(x - a)$$

$$= 6 + \frac{1}{12}(39 - 36)$$

$$= 6 + \frac{3}{12} = 6.25$$

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8 10 pts $a = 0, b = 4, \quad \frac{b-a}{n} = \frac{4}{n} = \Delta x$

$$x_k = a + k\Delta x = 0 + k \cdot \frac{4}{n} = \frac{4k}{n} \Longrightarrow$$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} (2x_k + 3) \cdot \frac{4}{n} =$$

$$= \frac{4}{n} \sum_{k=1}^{n} \left(2\left(\frac{4k}{n}\right) + 3\right) = \frac{4}{n}\left[\frac{8}{n}\sum_{k=1}^{n} k + 3\sum_{k=1}^{n} 1\right]$$

$$= \frac{4}{n} \cdot \frac{8}{n}\left(\frac{n^2 + \text{smaller}}{2}\right) + \frac{4}{n} \cdot 3 \cdot n$$

$$= \frac{32}{n^2}\left(\frac{n^2 + \sim}{2}\right) + 12$$

$$= 16\left(\frac{n^2 + \sim}{n^2}\right) + 12 \xrightarrow{n \to \infty} 16 + 12 = \boxed{28}$$

9 $$\int_0^4 (2x+3)\,dx = \left[x^2 + 3x\right]_0^4 = 4^2 + 3(4) - (0^2 + 3(0))$$

$$= 16 + 12 = \boxed{28}$$

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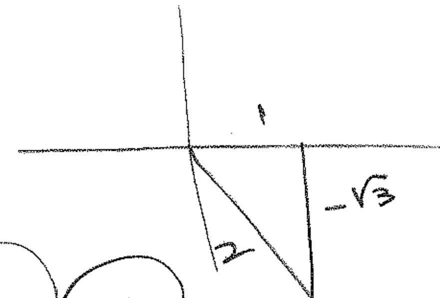
10 a 10pts

$$\int_0^{\frac{5\pi}{3}} \sin x \, dx = -\cos x \Big]_0^{\frac{5\pi}{3}}$$

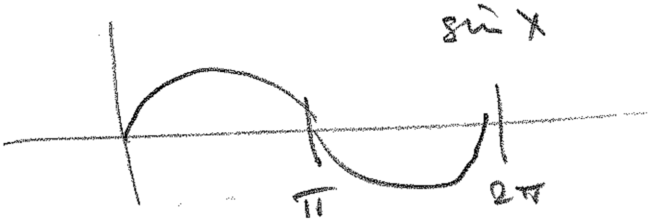


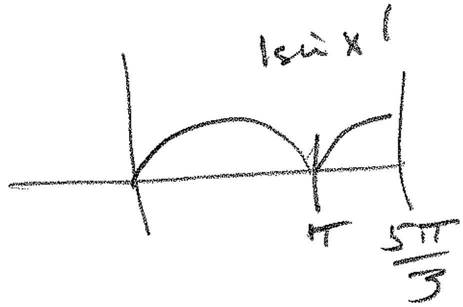
$$= -\cos\left(\tfrac{5\pi}{3}\right) - \left(-\cos(0)\right)$$

$$= -\left(\tfrac{1}{2}\right) + 1 = \tfrac{1}{2}$$

10pts Bonus 10b

$$\int_0^{\frac{5\pi}{3}} |\sin x \, dx| = \int_0^{\pi} \sin x \, dx - \int_{\pi}^{\frac{5\pi}{3}} \sin x \, dx$$





$$= -\cos x \Big]_0^{\pi} - \left(-\cos x\right)\Big]_{\pi}^{\frac{5\pi}{3}} = -\cos \pi - (-\cos 0)$$

$$+ \left[\cos\left(\tfrac{5\pi}{3}\right) - \cos \pi\right] = -0 + 1 + \tfrac{1}{2} - 0$$

$$= \boxed{\tfrac{3}{2}}$$

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10 c 10 pts $\displaystyle\int x^3 (x^4+2)^5\,dx$

$$= \frac{1}{4}\int (x^4+2)^5 (4x^3\,dx) = \frac{1}{4}\,\frac{(x^4+2)^6}{6} + C$$

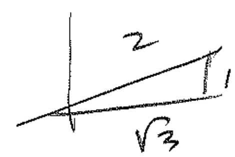
$$= \boxed{\frac{x^4+6}{24} + C}$$

10 d 10 pts $\displaystyle\int_{\frac{1}{6}}^{\frac{1}{2}} \csc(\pi t)\cot(\pi t)\,dt$

$u = \pi t \Rightarrow du = \pi\,dt \rightarrow$ Need factor of π inside of $\frac{1}{\pi}$ outside

$u(\tfrac{1}{6}) = \frac{\pi}{6}$, $u(\tfrac{1}{2}) = \frac{\pi}{2}$

$$= \frac{1}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc u \cot u\,du$$



$$= \frac{1}{\pi}\Big[-\csc u\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{1}{\pi}\Big[-\csc\frac{\pi}{2} - \big(-\csc\frac{\pi}{6}\big)\Big]$$

$$= \frac{1}{\pi}\big[-1+2\big] = \boxed{\frac{1}{\pi}}$$

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11 Arc Length $= L = \int_a^b \sqrt{1+f'(x)^2}\,dx$

$a=1,\ b=5$

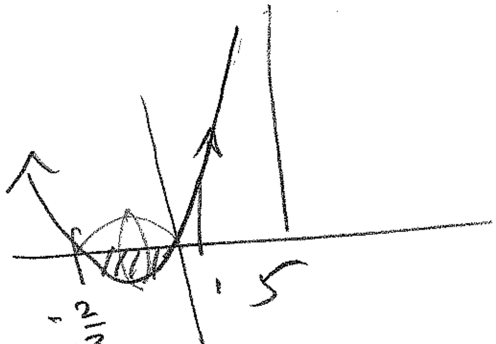
$f(x) = 3x^2+2x$

$f'(x) = 6x+2$

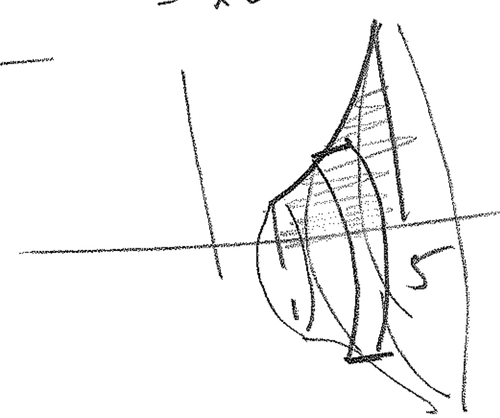
$f'(x)^2 = (6x+2)^2 = 36x^2+24x+4$

$$L = \int_1^5 \sqrt{1+(6x+2)^2}\,dx$$

12 a 10pts



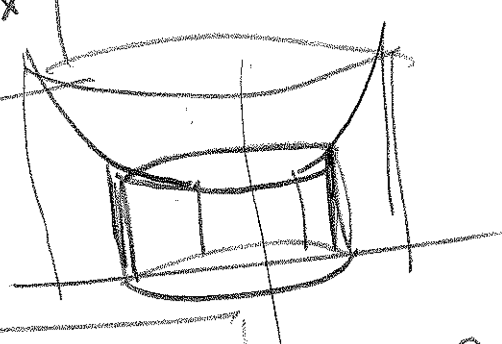
$3x^2+2x$
$= x(3x+2)$



x-axis

$$V = \pi\int_1^5 (3x^2+2x)^2\,dx$$

Discs



12b 10pts

$$\boxed{V = 2\pi\int_1^5 x\,(3x^2+2x)\,dx}$$

Shells

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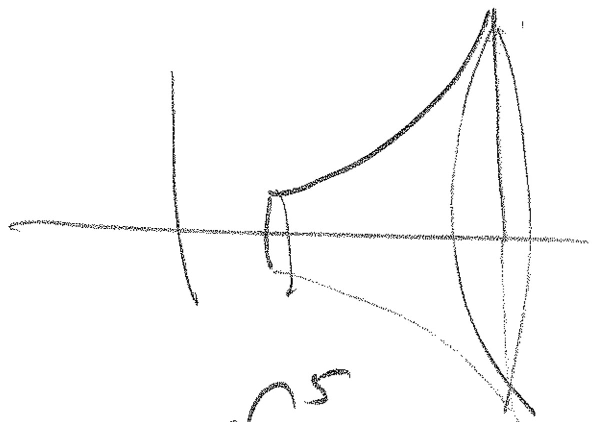

Surface area $\pi \int_1^5 f(x)\sqrt{1+f'(x)^2}\,dx$

$$= 2\pi \int_1^5 (3x^2+2x)\sqrt{1+(6x+2)^2}\,dx$$

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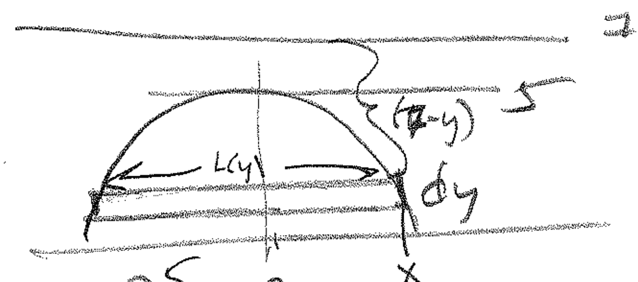

$x^2+y^2=5^2$

$x=\sqrt{25-y^2}$

$L(y)=2x$

$=2\sqrt{25-y^2}$

$$F = 62.4 \int_0^5 (7-y)(2\sqrt{25-y^2})\,dy$$