$$\int_{-\sqrt{7}}^{0} t\,(t^2+1)^{\frac{1}{3}}\,dt$$

$u = t^2+1$

$du = 2t\,dt$

$t=-\sqrt{7} \Rightarrow u = (-\sqrt{7})^2+1 = 8$

$t=0 \Rightarrow u = 0^2+1 = 1$

1

$$= \frac{1}{2}\int_{-\sqrt{7}}^{0} (t^2+1)^{\frac{1}{3}}(2t\,dt) = \frac{1}{2}\int_{8}^{1} u^{\frac{1}{3}}\,du = \frac{1}{2}\cdot\frac{3}{4}u^{\frac{4}{3}}\Big]_{8}^{1}$$

$$= \frac{3}{8}\left[1^{4/3} - 8^{4/3}\right] = \frac{3}{8}(-15) = -\frac{45}{8}$$

2

$$= \frac{1}{2}\left[\frac{(t^2+1)^{\frac{4}{3}}}{\frac{4}{3}}\right]_{-\sqrt{7}}^{0} = \frac{1}{2}\cdot\frac{3}{4}\left[(t^2+1)^{\frac{4}{3}}\right]_{-\sqrt{7}}^{0}$$

Scratch:

$$\left(8^{\frac{1}{3}}\right)^4 = 2^4 = 16$$

$$= \frac{3}{8}\left[(0^2+1)^{\frac{4}{3}} - ((-\sqrt{7})^2+1)\right]$$

$$= \frac{3}{8}\left[1^{\frac{4}{3}} - (7+1)^{\frac{4}{3}}\right]$$

$$= \frac{3}{8}\left[1 - 8^{\frac{4}{3}}\right] = \frac{3}{8}\left[1-16\right] = \frac{3}{8}\left[-15\right] = -\frac{45}{8}$$

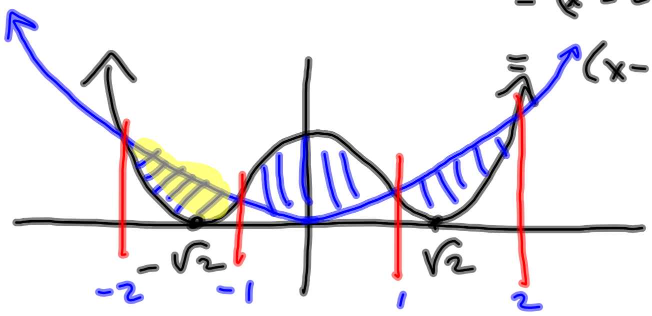
Area bdd by $f(x) = x^4 - 4x^2 + 4$ is EVEN: $f(-x) = f(x)$

& $g(x) = x^2$

$u = x^2 \Rightarrow u^2 - 4u + 4 = (u-2)^2$

$= (x^2 - 2)^2 = \left((x-\sqrt{2})(x+\sqrt{2})\right)^2$

$= (x-\sqrt{2})^2 (x+\sqrt{2})^2$



$f(x) = g(x) \longrightarrow f(x) - g(x) = 0$

$\Rightarrow x^4 - 4x^2 + 4 - x^2$

$= x^4 - 5x^2 + 4 \overset{\text{SET}}{=} 0$

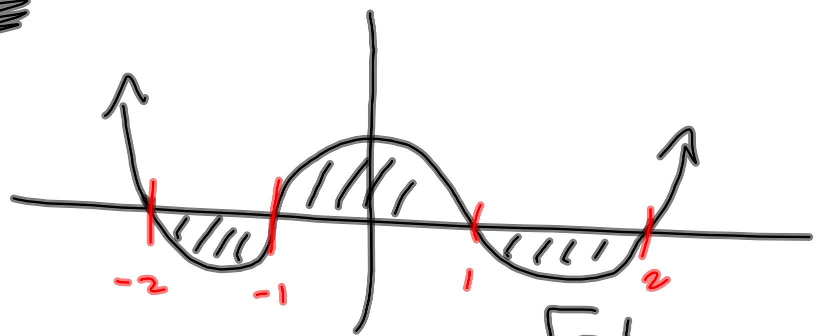
$u^2 - 5u + 4 = 0$

$(u-4)(u-1) = 0$

$(x^2-4)(x^2-1) = 0$

$\boxed{(x-2)(x+2)(x-1)(x+1)} = 0$

$f(x) - g(x) = \underline{x^4 - 5x^2 + 4}$



By symmetry: $A = 2\left[\int_0^1 f-g + \int_1^2 g-f\right]$ *

$= 2\int_0^2 |f-g|$

$= 2\left[\int_0^1 (x^4 - 5x^2+4)\,dx - \int_1^2 (x^4 - 5x^2+4)\,dx\right]$

$= 2\left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_0^1 - 2\left[\frac{x^5}{5} - \frac{5}{3}x^3 + 4x\right]_1^2$

$= 2\left[\frac{1}{5} - \frac{5}{3} + 4\right] - 2\left[\frac{32}{5} - \frac{40}{3} + 8 - \underline{\left(\frac{1}{5} - \frac{5}{3} + 4\right)}\right]$

$= 4\left[\frac{1}{5} - \frac{5}{3} + 4\right] - 2\left[\frac{32}{5} - \frac{40}{3} + 8\right]$ miami

~~$\int_0^2$~~ doesn't work. It misses when the x^2 jumps above the $x^4 - 5x^2 + 4$

= etc. See Sol'ns.

Solutions to everything but §6.5 are posted.

1 10 pts - Σ

10 pts - FTC

2 f_{avg} = 10 pts

3 $c \ni f(c) = f_{avg}$ 10 pts

4 a 15 pts

b 15 pts

c 15 pts

5 15 pts

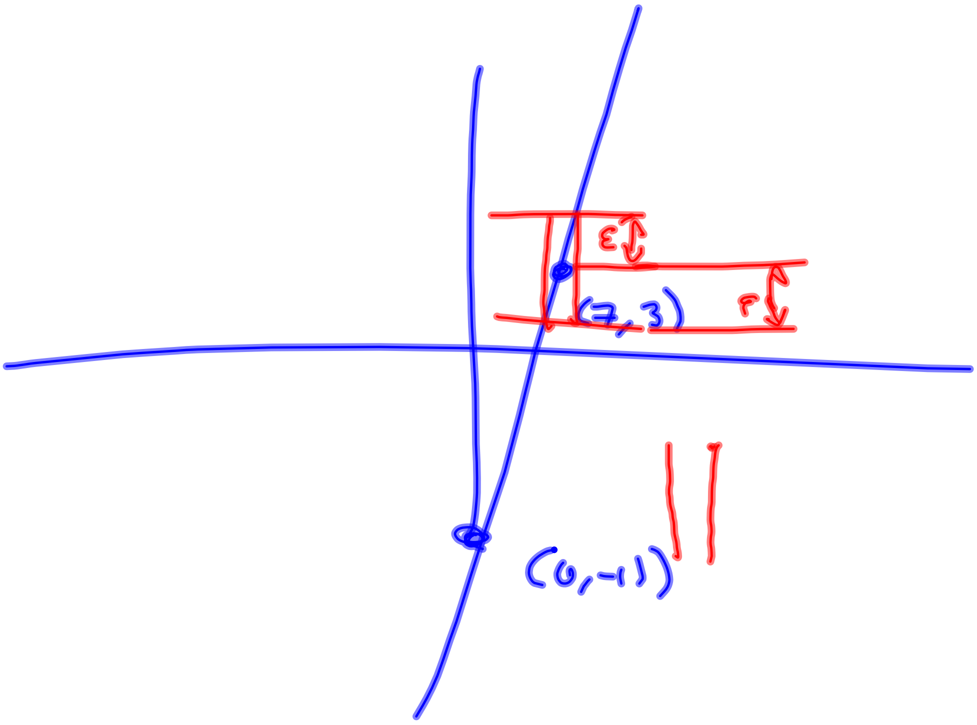
Dec 3-12:36 PM

$$\lim_{x\to 7}(2x-11) = 3$$

Pf Let $\varepsilon > 0$ be given

Define $\delta = \frac{\varepsilon}{2}$. Then, if $0 < |x-7| < \delta$, we have

$$|(2x-11)-3| = |2x-14| = 2|x-7| < 2\delta = 2\cdot\frac{\varepsilon}{2} = \varepsilon \ \blacksquare$$



Prove $f(x) = 2x - 11$ is continuous

for all $x \in \mathbb{R}$.

want to show $\lim_{x \to c} f(x) = f(c)$, regardless

of our choice of c.

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{2}$

Then $f(c) = 2c - 11$ and

if $0 < |x - c| < \delta$, we have

$|(2x-11) - (2c-11)| = |2x - 2c| = 2|x-c| < 2\delta$

$= \varepsilon \Rightarrow \lim_{x \to c} (2x-11) = 2c - 11 = f(c)$ $\blacksquare$

$$\int_1^5 (x^2+3x-5)\,dx$$

$$\Delta x = \frac{5-1}{n} = \frac{4}{n}$$

$$x_k = 1 + k\Delta x = 1 + \frac{4k}{n} = \frac{4k}{n} + 1$$

$$\sum f(x_k)\,\Delta x = \sum\left(\left(\frac{4k}{n}+1\right)^2 + 3\left(\frac{4k}{n}+1\right) - 5\right)\left(\frac{4}{n}\right)$$

$$= \frac{4}{n}\left[\sum\left(\frac{16k^2}{n^2} + \frac{8k}{n} + 1 + \frac{12k}{n} + 3 - 5\right)\right]$$

$$= \frac{4}{n}\left[\frac{16}{n^2}\sum_{k=1}^{n} k^2 + \frac{20}{n}\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1\right]$$

$$= \frac{4}{n}\left[\frac{16}{n^2}\cdot\frac{n^3+\cdots}{3} + \frac{20}{n}\cdot\frac{n^2+\cdots}{2} - n\right]$$

$$\xrightarrow{n\to\infty} \frac{4}{n}\cdot\frac{16}{n^2}\cdot\frac{n^3}{3} + \frac{4}{n}\cdot\frac{20}{n}\cdot\frac{n^2}{2} + \frac{4}{n}(-n) \quad \sum$$

$$= \frac{64}{3} + 40 - 4 = \frac{64+120-12}{3} = \frac{172}{3}$$

FTC:

$$\int_1^5 (x^2+3x-5)\,dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} - 5x\right]_1^5$$

$$= \frac{125}{3} + \frac{75}{2} - 25 - \left(\frac{1}{3} + \frac{3}{2} - 5\right) \qquad 970-290-0$$

$$= \frac{124}{3} + \frac{72}{2} - 20 = \frac{124}{3} + 36 - 20$$

$$= \frac{124}{3} + 16$$

$$= \frac{124}{3} + \frac{48}{3} = \frac{172}{3}$$

$$\int t\,(t^2+1)^{\frac{1}{3}}dt$$

$u = t^2+1$

$du = 2t\,dt$

$dt = \dfrac{du}{2t}$

$$= \int \cancel{t}\,u^{\frac{1}{3}}\,\frac{du}{2\cancel{t}}$$

$$= \frac{1}{2}\int u^{\frac{1}{3}}du$$

Dec 3-1:05 PM