

(a) Not cont^l @ $x = -1$
 $\lim_{x \rightarrow -1} f(x) = 1 \neq -1 = f(-1)$

(b) No. $\lim_{x \rightarrow 3^-} f(x) = 3 \neq -1 = \lim_{x \rightarrow 3^+} f(x)$

(c) $\lim_{x \rightarrow -1} f(x) = 1$

(d) Define $f(-1) = 1$ to make it cont^l @ $x = -1$.

Prove $\lim_{x \rightarrow 3} (3x+7) = 16$.

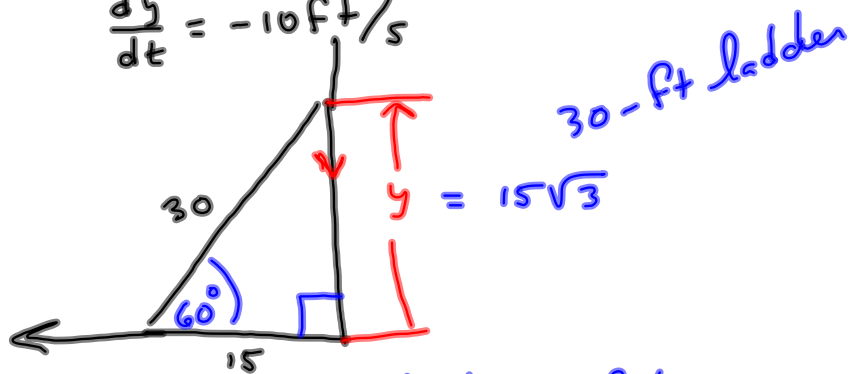
Proof Let $\epsilon > 0$ be given. Define $\delta = \frac{\epsilon}{3}$.

Then $0 < |x-3| < \delta$ implies

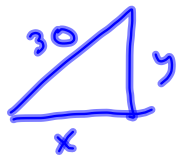
$$|(3x+7) - 16| = |3x-9| = 3|x-3| < 3\delta = 3 \cdot \frac{\epsilon}{3} = \epsilon \quad \square$$

Ladder falling

$$\frac{dy}{dt} = -10 \text{ ft/s}$$



How fast is the bottom sliding away from the wall, when it's 15 feet away?



want $\frac{dx}{dt}$, given $\frac{dy}{dt} = -10$

$$x^2 + y^2 = 30^2$$

$$y(x(t)) = y = \sqrt{30^2 - x^2} = y(x(t)) = \sqrt{30^2 - (x(t))^2}$$

$$\frac{dy}{dt} = \frac{1}{2} (900 - x^2)^{-\frac{1}{2}} (-2x) \frac{dx}{dt}$$

$\frac{dy}{dx}$

Chain rule says $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$$\left. \frac{dy}{dt} \right|_{x=15} = \frac{1}{2} (900 - 15^2)^{-\frac{1}{2}} (2(15)) \left. \frac{dx}{dt} \right|_{x=15} = -10$$

$$= -\frac{1}{2} \left(\frac{1}{\sqrt{675}} \right) (30) \left. \frac{dx}{dt} \right|_{x=15} = -10$$

$$\underline{= -15} \left(\frac{1}{15\sqrt{3}} \right) \left. \frac{dx}{dt} \right|_{x=15} = -10$$

$$\begin{array}{r} 3 \overline{) 675} \\ \underline{330} \\ 345 \\ \underline{330} \\ 15 \end{array}$$

$$\frac{1}{\sqrt{3}} \frac{dx}{dt} = +10$$

$$\frac{dx}{dt} = +10\sqrt{3}$$

Implicit Differentiation.

$$\frac{d}{dt} [x^2 + y^2 = 30^2]$$

$$2xx' + 2yy' = 0$$

$$xx' = -yy'$$

$$x' = \frac{-yy'}{x} = \frac{-15\sqrt{3} \cdot (-10)}{15} = 10\sqrt{3}$$

$$x' = \frac{dx}{dt}$$

$$y' = \frac{dy}{dt}$$

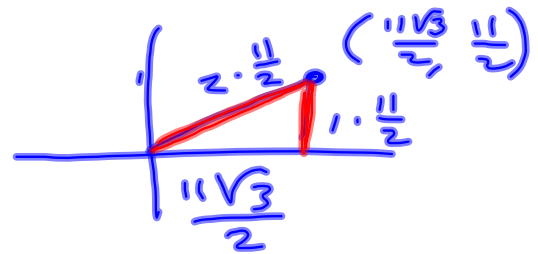
Find an equation of the tangent
to a circle of radius 11 @ $(\frac{11\sqrt{3}}{2}, \frac{11}{2})$

$$\frac{d}{dx} [x^2 + y^2 = 11^2]$$

$$2x + 2y(y') = 0$$

$$y' = -\frac{x}{y}$$

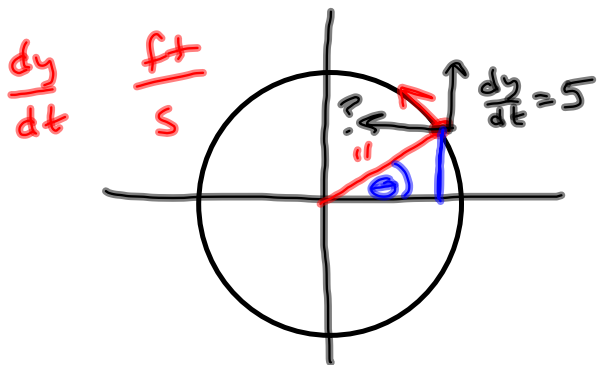
$$= -\frac{\frac{11\sqrt{3}}{2}}{\frac{11}{2}} = -\frac{11\sqrt{3}}{11} = -\sqrt{3}$$



$$y = -\sqrt{3} \left(x - \frac{11\sqrt{3}}{2} \right) + \frac{11}{2}$$

$$y = f'(a)(x-a) + f(a)$$

$$\frac{f(x+h) - f(x)}{h} \quad y = \frac{dy}{dx} \Big|_{x=a} (x-a) + y \Big|_{x=a}$$



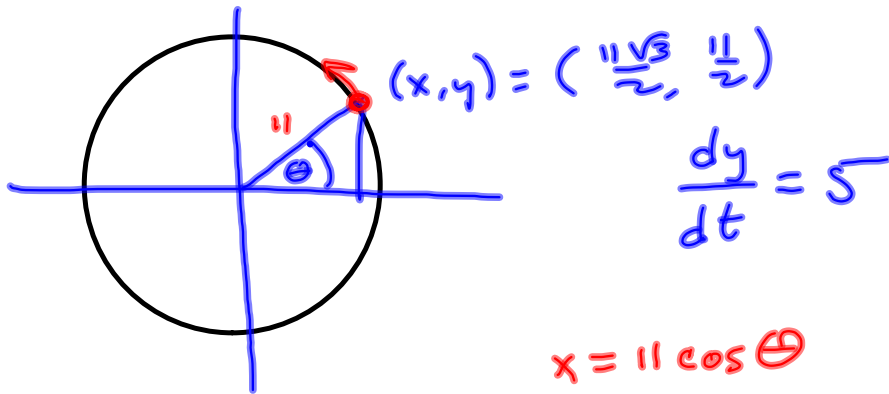
When $(x, y) = \left(\frac{1\sqrt{3}}{2}, \frac{1}{2}\right)$,
 we know that
 $\frac{dy}{dt} = 5$. Find the
 rate at which it's
 moving to the left.

$$\frac{d}{dt} [x^2 + y^2 = 11^2]$$

$$2x x' + 2y y' = 0$$

$$\frac{dx}{dt} = x' = -\frac{y}{x} y'$$

$$= -\frac{\frac{1}{2}}{\frac{1\sqrt{3}}{2}} \cdot 5 = 5\sqrt{3}$$



$$\frac{y}{11} = \sin \theta$$

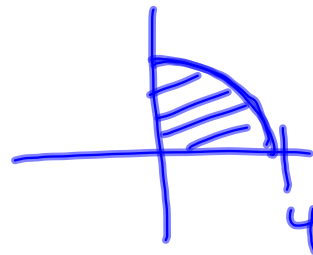
$$y = 11 \sin \theta$$

$$\frac{dy}{dt} = -11 \cos \theta \frac{d\theta}{dt}$$

$$\frac{5}{-11} = \frac{d\theta}{dt}$$

$$\frac{dx}{dt} = 11 \sin \theta \frac{d\theta}{dt}$$

$$\int_0^4 \sqrt{16-x^2} dx$$



$$y = \sqrt{16-x^2}$$

$$x^2 + y^2 = 16 = 4^2$$