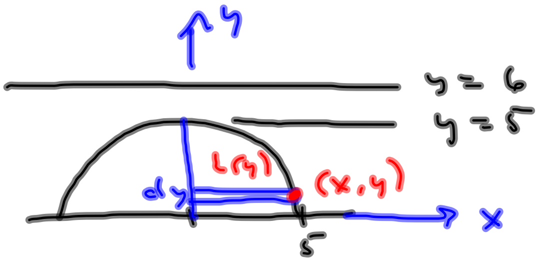


$62.4\, d \cdot L(y)$

$d = 6 - y$

$x^2 + y^2 = 5^2$

$L(y)^2 + y^2 = 5^2$

$L(y) = \sqrt{25 - y^2}$

$$62.4 \int_0^5 (6-y)\sqrt{25-y^2}\, dy$$

$$62.4 \left[6 \int_0^5 \sqrt{25-y^2}\, dy + \int_0^5 -y\sqrt{25-y^2}\, dy \right]$$

$$y = \int_0^x \sqrt{\sec^2 t - 1}\, dt \qquad \int \frac{f'(x)\,dx}{f(x)}$$

$$= \ln|f(x)| + C$$

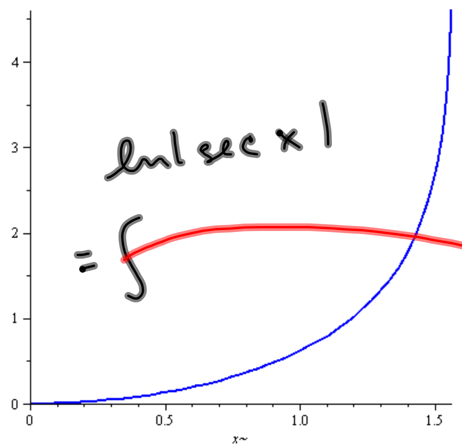
$$= \int_0^x \tan t \, dt \qquad -\int \frac{-\sin t\, dt}{\cos t} =$$

$$-\ln|\cos t| + C$$

$$ds = \sqrt{1+(y')^2}\, dt \qquad = \ln\left(|\cos t|^{-1}\right) + C$$

$$y'(x) = \sqrt{\sec^2 x - 1} = \tan x \qquad = \ln|\sec t| + C$$

y'



$\ln|\sec x|$

$= \int$

Plot for #2 on §6.3 assignment.

Graph of

$$\int_0^x \sqrt{\sec^2 t - 1}\, dt$$

$$= \ln|\sec x| - \ln|\sec 0|$$

$$= \ln|\sec x| - \ln 1$$

$$= \ln|\sec x|$$

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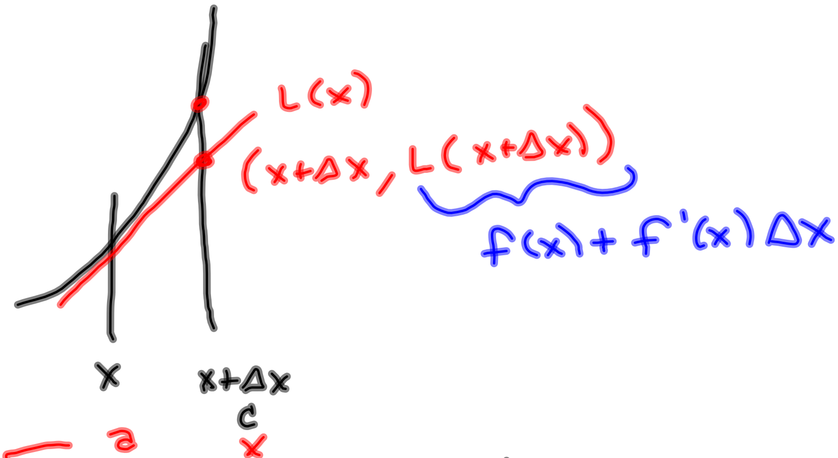
$$f(x+\Delta x) \approx f(x) + f'(x)\Delta x$$



$$L(c) = f(x) + f'(x)(x-c)$$

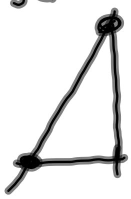
$$L(x) = f(a) + f'(a)\underbrace{(x-a)}_{\Delta x}$$

$$\sin(65^\circ) = \sin(60^\circ + 5^\circ)$$

$f(x) = \sin x$ $\qquad a = 60^\circ = \frac{\pi}{3}$ rads

$\Delta x = 5^\circ = \frac{5\pi}{180} = \frac{\pi}{36}$

$f'(x) = \cos x$



$$f(a + \Delta x) \approx f(a) + f'(a)\Delta x$$

$$= \sin\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right) \cdot \frac{\pi}{36}$$

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{72}$$

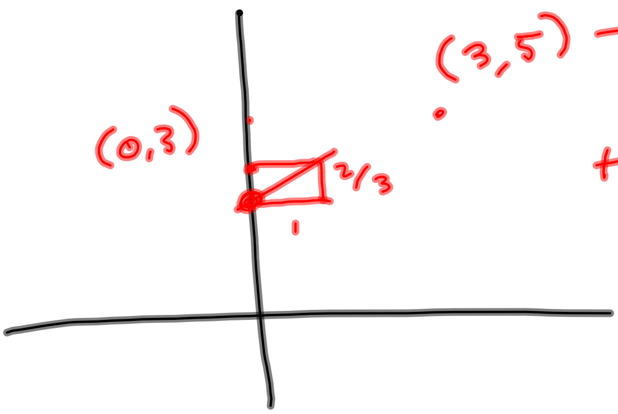
Δx

$$L(x) = f(a) + f'(a)(x - a)$$

$$= \sin\left(\frac{\pi}{3}\right) + \left(\cos\left(\frac{\pi}{3}\right)\right)\underbrace{(65^\circ - 60^\circ)}_{\frac{\pi}{36} = 5^\circ}$$

$$=$$

$y = \frac{2}{3}x + 3$



$(3,5) \longrightarrow$ 3 units to the right

times $\frac{2}{3}$: $3 \cdot \frac{2}{3} = 2$

up 2

$P(n)$: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

Let S' = the set of all natural #s n $\ni$ the statement $P(n)$ holds

Proof

Clearly $\sum_{k=1}^{1} k = 1 = \frac{1(1+1)}{2}$ So $1 \in S' \neq \emptyset$.

Suppose $P(n)$ holds for some $n \geq 1$.

Then $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ and

$$\sum_{k=1}^{n+1} k = \boxed{\underbrace{1+2+3+\cdots+(n-1)+n}_{\sum_{k=1}^{n} k = \boxed{\frac{n(n+1)}{2}}}} + (n+1)$$

$$= \frac{n(n+1)}{2} + n+1$$

$$= \frac{n^2+n}{2} + \frac{2(n+1)}{2} = \frac{n^2+n+2n+2}{2}$$

$$= \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2} = \frac{(n+1)((n+1)+1)}{2}$$

$\Longrightarrow$ it's the same for $n+1$!

$P(n): \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

---

① Let $S = \{ n \mid P(n) \text{ holds} \}$

② Show $P(1)$ holds, so $1 \in S$ & $S \neq \emptyset$

③ Assume $P(n)$ holds for $n \geq 1$.

④ Use $P(n)$ to get $P(n+1)$ to hold

$P(n+1): \sum_{k=1}^{n+1} k = \boxed{\frac{(n+1)((n+1)+1)}{2}}$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Want to get

$$\sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2(n+1)+1)}{6}$$

$$\sum_{k=1}^{1} k^2 = 1^2 = 1 \overset{?}{=} \frac{1(1+1)(2(1)+1)}{6} = \frac{(2)(3)}{6} = 1 \checkmark$$

So $1 \in S$. Let $n \geq 1 \ni n \in S$. Then

$$\sum_{k=1}^{n+1} k^2 = \sum_{k=1}^{n} k^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{6(n+1)^2}{6}$$

$$= \frac{[n+1][n(2n+1) + 6(n+1)]}{6}$$

$$= \frac{[n+1][2n^2 + n + 6n + 6]}{6}$$

$$= \frac{[n+1][2n^2 + 7n + 6]}{6} = \frac{(n+1)(2n^2 + 4n + 3n + 6)}{6}$$

$$= \frac{(n+1)(2n(n+2) + 3(n+2))}{6}$$

$2(n+1)+1$

$$= \frac{(n+1)(n+2)(2n+3)}{6}$$

$$= \frac{(n+1)((n+1)+1)(2(n+1)+1)}{6} \quad \blacksquare$$

$n+1 \in S$ $\blacksquare$ $P(n+1) \checkmark$

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