Final 12:10–2:00 Thursday

Use differentials or tangent line to estimate $\sqrt[3]{24}$

Use ~~same idea~~ differentials to estimate the amount of paint to cover the sides of a water tank that's a cylinder of radius 10 ft, height 10 ft, if a coat of paint is $\frac{1}{16}$" thick.

Nov 27-12:05 PM

Use differentials on tangent line to estimate $\sqrt[3]{24}$

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$a = 27 \qquad \Delta x = dx = 24 - 27 = -3$$

$$f(a) = \sqrt[3]{27} = 3$$

$$f'(x) = \frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\left(x^{\frac{1}{3}}\right)^2}$$

$$f'(a) = \frac{1}{3\left(27^{\frac{1}{3}}\right)^2} = \frac{1}{3(3)^2} = \frac{1}{27}$$

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(a+\Delta x) \approx f(a) + f'(a)\Delta x$$

$$y = f(27) + f'(27)(x-27)$$

$$= 3 + \left(\frac{1}{27}\right)(-3)$$

$$= 3 - \frac{1}{9}$$

$$= \boxed{\frac{26}{9}} \text{ OR } 2\frac{8}{9} = 2.\overline{88}$$

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26/9
        2.888888889
24^(1/3)
        2.884499141
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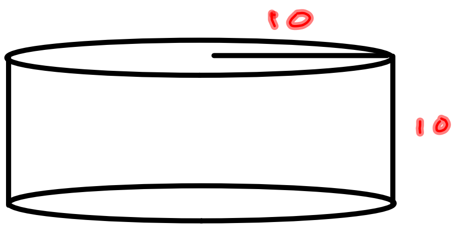
$\sqrt[3]{24} \approx 2.884499141$, by calculator

Accurate to one digit to right of decimal.

differentials

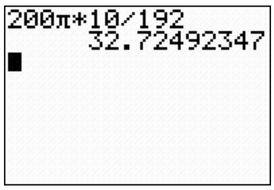
Use same idea to estimate the amount of paint to cover the sides of a water tank that's a cylinder of radius 10 ft, height 10 ft, if a coat of paint is $\frac{1}{16}$" thick.



Volume $= V = \pi r^2 h = 10\pi r^2$, since $h=10$ is fixed

$$\frac{dV}{dr} = 20\pi r \Longrightarrow$$

$$\Delta V \approx dV = 20\pi r \, dr \Longrightarrow$$



$$dV\Big|_{\substack{r=10 \\ dr=\frac{1}{192}}} = 20\cdot\pi(10)\left(\frac{1}{192}\right) \approx 3.272492347 \text{ ft}^3$$

$$\left(\frac{1}{16}\text{ in}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \frac{1}{192}\text{ ft}$$

1 Gallon [Fluid, US] = 0.133680556 Cubic Feet
1 Gallon [Dry, US] = 0.155557003 Cubic Feet

$$(3.272492347\ ft^3)\left(\frac{1\ gal}{.33680556\ ft^3}\right)$$

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200π*10/192
          32.72492347
Ans/.33680556
          97.16265811
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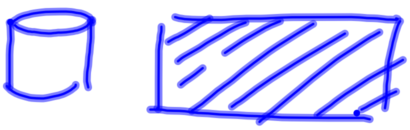
off by factor of 10

$$\approx 9.716265811$$



Check: surface area is $2\pi rh$

$= 2\pi(10)(10) = \boxed{200\pi \approx 628\ ft^2}$

multiply by thickness of $\frac{1}{16}$"

$$(628)\left(\frac{1}{192}\right) \approx (3.27\ ft^3)\left(\frac{1\ gall}{.33680556\ ft^3}\right)$$

$$\approx 9.72\ gals$$

$$\frac{d}{dx}\left[\frac{\sin^2(x^2-5x)}{(2x+3)^{1/2}}\right] \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\left[f(g(h(x)))\right] = \frac{df}{dg}\cdot\frac{dg}{dh}\cdot\frac{dh}{dx} \qquad \frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$$= \frac{\overbrace{\overbrace{(2\sin(x^2-5x))}^{\frac{df}{dg}}\overbrace{(\cos(x^2-5x))}^{\frac{dg}{dh}}\overbrace{(2x-5)}^{\frac{dh}{dx}}}^{f'}\overbrace{(2x+3)^{\frac{1}{2}}}^{g} - \overbrace{\sin^2(x^2-5x)}^{f}\overbrace{\left(\tfrac{1}{2}(2x+3)^{-\frac{1}{2}}(2)\right)}^{g'}}{\underbrace{2x+3}_{g^2}}$$

$$\frac{d}{dx}\left[(2x+3)^{\frac{1}{2}}\right] = \frac{1}{2}(2x+3)^{-\frac{1}{2}}(2)$$