§6.5 #s 6, 8, 11, 14, 34

12:10 – 2:00 Thursday FINAL EXAM.

WORK = FORCE • DISTANCE

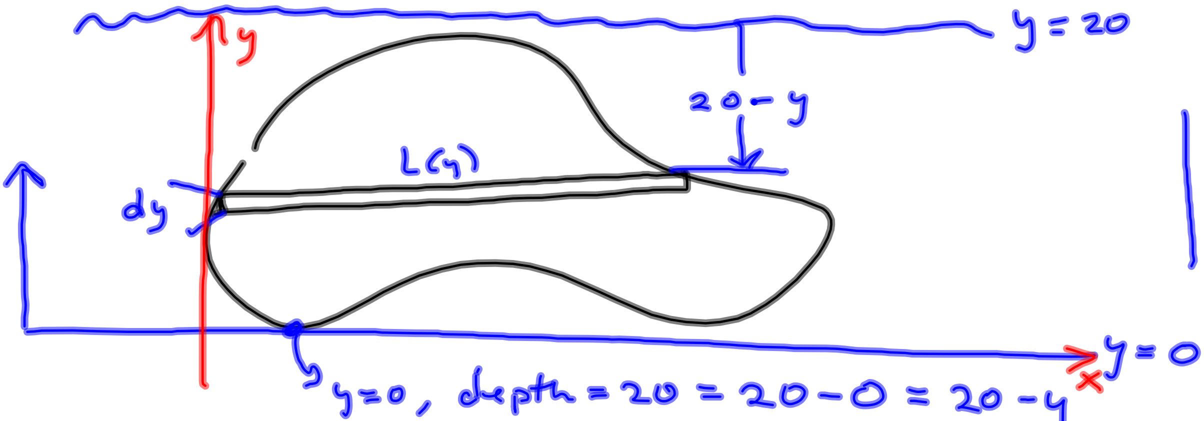
$$= \int_a^b \underbrace{F(x)}_{\text{Force (variable)}} \underbrace{dx}_{\text{Incremental distance}}$$

Fluid Force on a vertical plate.

w = weight density

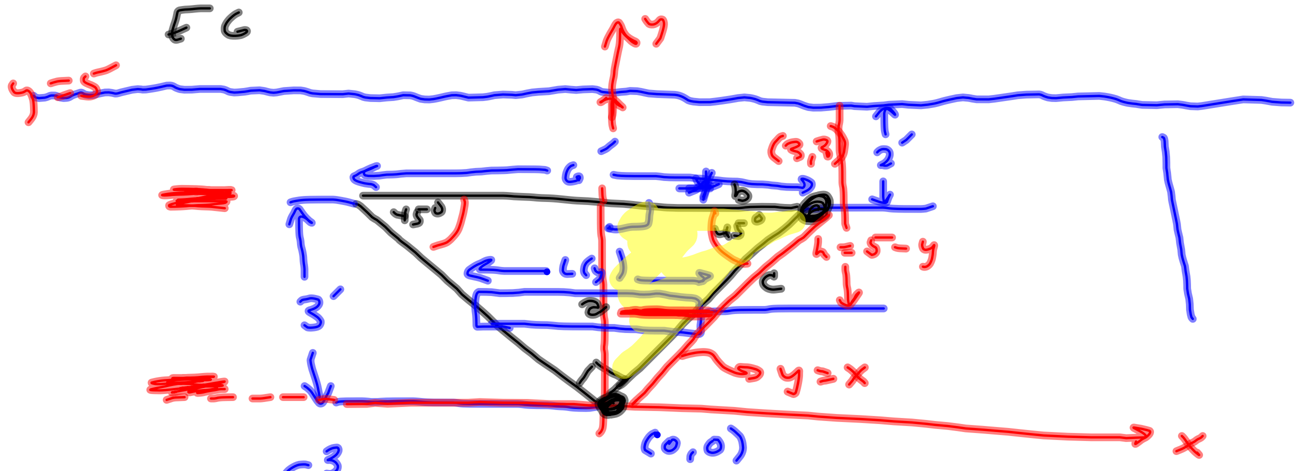
pressure = w · h, where h = depth.

depth measures vertically down



$$\int_0^b \underbrace{w}_{62.4\ \text{lb}/\text{ft}^3} \cdot \underbrace{(\text{strip depth})}_{20-y} \underbrace{L(y)}_{x = L(y)} dy$$

EG



$$\int_0^3 62.4\,(5-y)\,L(y)\,dy$$

x

Find eq'n for $x = L(y)$:

$y = f(x)$

$y = m(x - x_1) + y_1$

$= 1(x-0)+0$

$y = x$

By symmetry

$L(y) = 2x = 2y$

$$F = \int_0^3 62.4\,(5-y)(2y)\,dy \quad \text{etc.}$$

$$= 124.8\int (5y - y^2)\,dy \quad \text{etc.}$$

How to get m:

$\frac{a}{b} = \tan 45^\circ$

$\frac{3}{b} = \tan 45^\circ = m$

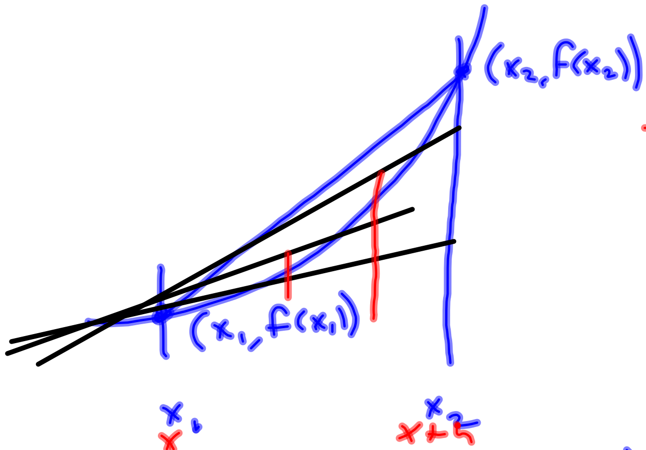
$\frac{3}{\tan 45^\circ} = b$

$3\cot 45^\circ = b$

$3 = b$

$\frac{3}{3} = \tan 45^\circ = 1 = m$

Differential Calculus - We generalize the notion of slope of a straight line to slope of a curve.



Slope @ $x_1 \approx \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} \xrightarrow{x_2 \to x_1} f'(x_1)$

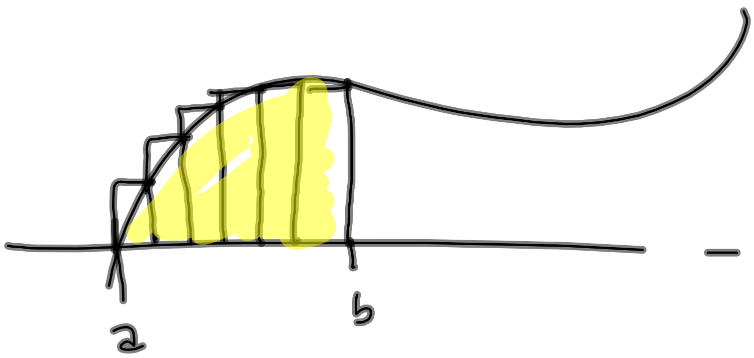
= Slope @ x_1 exactly, provided the limit exists.

f must be smooth.

Average Slope = Slope of Secant Line

$= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(x+h) - f(x)}{h}$

Integral Calculus



Area ≈ sum of the rectangles

$$= \sum_{k=1}^{n} f(x_k)\,\Delta x_k$$

$$\xrightarrow{\Delta x_k \to 0} \int_a^b f(x)\,dx$$

This forces $n \to \infty$

And if rectangles are equal width, then $n \to \infty$ <u>implies</u> $\Delta x_k \to 0$.

But $n \to \infty$ DOES NOT imply $\Delta x \to 0$, without equal widths or other condition forcing $\Delta x \to 0$.

Need $f(x)$ is continuous. (or piecewise continuous)

(Discontinuous on at most a set of measure zero.)

$$\lim_{x \to 3} (2x-7) = -1 \text{ means...}$$

Challenge: Give me an $\varepsilon > 0$.

Response: I can find a $\delta > 0$ so that any time the distance from x to 3 (without $x=3$) is less than δ, I can guarantee $f(x)$ is closer to -1 than ε.

$\varepsilon > 0 \Rightarrow$

$\exists\, \delta > 0 \ni 0 < |x-3| < \delta \Rightarrow |f(x) - (-1)| < \varepsilon$

Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{2}$. Then $0 < |x-3| < \delta$

$\Rightarrow |2x-7-(-1)| = |2x-6| = 2|x-3|$

$< 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon$ $\blacksquare$