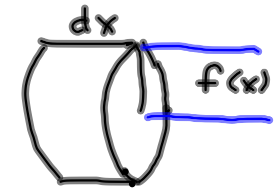
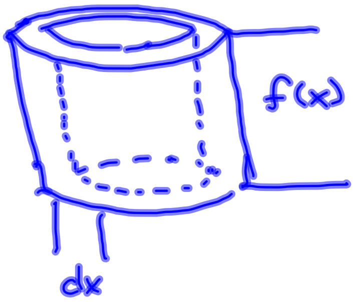
Volume :

Disks $\pi \int_a^b f(x)^2 \, dx$



Shells $2\pi \int_a^b x \, f(x) dx$



Surface area

$$2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} \, dx$$ about x-axis

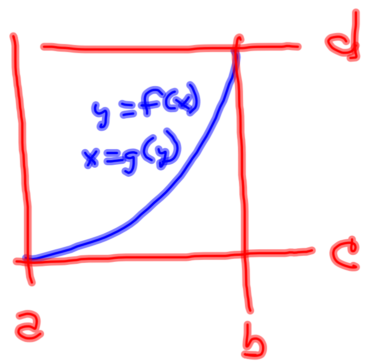
$\sqrt{1 + (f'(x))^2} \, dx$, $\sqrt{(dx)^2 + (dy)^2}$, $\sqrt{(g'(y))^2 + 1} \, dy$ → ds

If

$$= 2\pi \int_a^b y \, ds$$

6.4 #22 is a nice exploration of this.

Depends on what's easier



$$2\pi \int_a^b f(x) \, ds =$$

ds

$$2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} \, dx$$

I'm not expressing this well.

$$2\pi \int_c^d y \sqrt{1 + (g'(y))^2} \, dy$$

Look @ #22 §6.4 as separate Bonus Assignment.

Nov 15-12:12 PM

Like #6 in §6.4

$y = x + 2\sqrt{x}$ about x-axis $1 \le x \le 2$

$y' = 1 + \frac{1}{\sqrt{x}}$

$(y')^2 = 1 + \frac{2}{\sqrt{x}} + \frac{1}{x}$

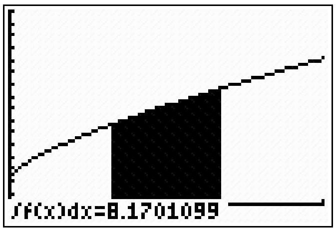
Use tech.

$(y')^2 + 1 =$

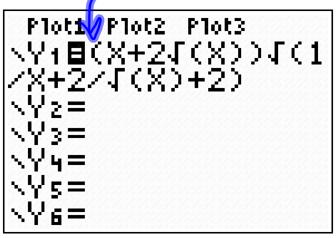
$\frac{1}{x} + \frac{2}{\sqrt{x}} + 2$

$$2\pi \int_1^2 f(x)\sqrt{1+(f'(x))^2}\,dx = 2\pi \int_1^2 (x+2\sqrt{x})\sqrt{\frac{1}{x} + \frac{2}{\sqrt{x}} + 2}\,dx$$

Very difficult to evaluate.



TIMES 2π (FORGOT IT)



Forgot the 2π

§6.4 #6 is doable.

$y = \sqrt{x+1}$, $1 \le x \le 5$, x-axis.

$y' = \frac{1}{2\sqrt{x+1}}$

$(y')^2 = \frac{1}{4(x+1)}$

$(y')^2 + 1 = \frac{1}{4(x+1)} + 1$

$$\text{Area} = 2\pi \int_1^5 \sqrt{x+1}\sqrt{1 + \frac{1}{4(x+1)}}\, dx$$

$a^b c^b = (ac)^b$

$(ac)^b = a^b c^b$

$$= 2\pi \int_1^5 \sqrt{x+1+\frac{1}{4}}\, dx$$

$\int u^{\frac{1}{2}} du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$$= 2\pi \int_1^5 \sqrt{x+\frac{5}{4}}\, dx = 2\pi \int_1^5 \left(x+\frac{5}{4}\right)^{\frac{1}{2}} dx$$

$u = x + \frac{5}{4}$

$du = dx$

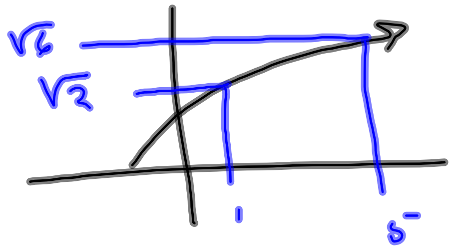
$$= \left[2\pi \cdot \frac{2}{3}\left(x+\frac{5}{4}\right)^{\frac{3}{2}}\right]_1^5$$

$$= \frac{4\pi}{3}\left[\left(x+\frac{5}{4}\right)^{\frac{3}{2}}\right]_1^5 = \frac{4\pi}{3}\left[\left(\frac{25}{4}\right)^{\frac{3}{2}} - \left(\frac{9}{4}\right)^{\frac{3}{2}}\right]$$

$$= \frac{4\pi}{3}\left[\frac{125}{8} - \frac{27}{8}\right] = \frac{4\pi}{3}\left[\frac{98}{8}\right] = \frac{4\pi}{3}\left[\frac{49}{4}\right]$$

$$= \frac{49\pi}{3}$$

$y = \sqrt{x+1}$, $1 \le x \le 5$, x-axis.



$y = \sqrt{x+1}$

$y^2 = x+1$

$x = y^2 - 1$

$\frac{dx}{dy} = 2y = x'$

$2\pi \int_1^5 f(x)\sqrt{1+f'(x)^2}\,dx$ $\quad (x')^2 = (2y)^2 = 4y^2$

$= 2\pi \int_{\sqrt{2}}^{\sqrt{6}} y\sqrt{1+4y^2}\,dy$

$2\pi \int \underbrace{f(x)}_{y} \underbrace{\sqrt{1+(f'(x))^2}\,dx}_{ds}$

$$2\cdot \mathrm{Pi} \int_1^5 \sqrt{x+1}\cdot\sqrt{1+\frac{1}{4\cdot(x+1)}}\,dx$$

$$\frac{49}{3}\pi$$

$$2\cdot \mathrm{Pi}\cdot \int_{\sqrt{2}}^{\sqrt{6}} y\sqrt{1+4\cdot y^2}\,dy$$

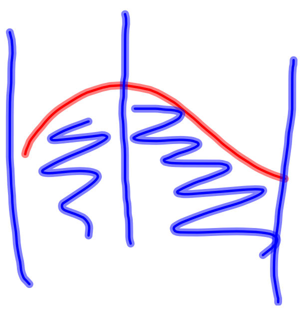
$$\frac{49}{3}\pi$$

SAME!

#22 is getting at, if I'm not mistaken

iinm

AFAIK



$$\frac{d}{dx}\left[(x^5-7)^{34} \right]$$

$$= 34(x^5-7)^{33}(5x)$$

$$\frac{d}{dx}\left[\sec^2(x^2-3x) \right]$$

$$= \frac{d}{dx}\left[\left(\sec(x^2-3x)\right)^2 \right]$$

$$= \underbrace{2\sec(x^2-3x)^1}_{\frac{df}{dg}} \underbrace{\left(\sec(x^2-3x)\tan(x^2-3x)\right)}_{\frac{dg}{dh}} \underbrace{(2x-3)}_{\frac{dh}{dx}}$$

$f(u) = u^2$ $\quad \frac{df}{du} = 2u$

$g(v) = \sec(v)$ $\quad \frac{dg}{dv} = \sec v \tan v$

$h(x) = x^2-3x$ $\quad \frac{dh}{dx} = 2x-3$

$$\frac{d}{dx}\left[f(g(h(x))) \right] = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dx}$$

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$$\frac{d}{dx}\left[\left(\sec(x^2-3x)\right)^2\right] =$$

$$\frac{d\left(\left(\sec(x^2-3x)\right)^2\right)}{d\left(\sec(x^2-3x)\right)} \cdot \frac{d\left(\sec(x^2-3x)\right)}{d(x^2-3x)} \cdot \frac{d(x^2-3x)}{dx}$$

$$\left(2\sec(x^2-3x)\right)\left(\sec(x^2-3x)\tan(x^2-3x)\right)(2x-3)$$

$f(u) = u^2$ $\qquad \frac{df}{du} = 2u = 2\sec(x^2-3x)$

$u = \sec(x^2-3x)$

$g(v) = \sec(v)$ $\qquad \frac{dg}{dv} = \sec(v)\tan(v)$

$v = x^2-3x$ $\qquad = \sec(x^2-3x)\tan(x^2-3x)$