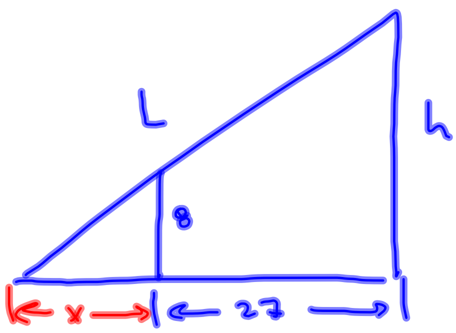


$L = \sqrt{(x+27)^2 + h^2}$

To minimize L, minimize

$L^2 = (x+27)^2 + h^2$

Similar triangles

$\frac{h}{x+27} = \frac{8}{x} \Rightarrow h = \frac{8}{x}(x+27)$

$= 8\left(1+\frac{27}{x}\right)$

$\Rightarrow L^2 = f(x) = (x+27)^2 + 8^2\left(\frac{x+27}{x}\right)^2 = 64\left(1+\frac{27}{x}\right)^2$

$\Rightarrow f'(x) = 2(x+27) + 64(2)\left(1+\frac{27}{x}\right)\left(-\frac{27}{x^2}\right)$

$= 2(x+27) + 8^2 \cdot 2\left(\frac{x+27}{x}\right)\left(-\frac{27}{x^2}\right) \overset{\text{set}}{=} 0$

$\Rightarrow 2(x+27)\left[1 + 8^2\left(-\frac{27}{x^3}\right)\right] = 0$

$\Rightarrow (x+27)\left[\frac{x^3 + 64(-27)}{x^3}\right] = 0$

$\Rightarrow (x+27)(x^3 - (64)(27)) = 0$

$\cancel{x = -27}$ OR $x = \sqrt[3]{64(27)}$

$= \sqrt[3]{2^6 \cdot 3^3}$

$= 2^2 \cdot 3^1 = 12 = x$

2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2

3 | 27
3 | 9
3

So, $x+27 = 39$ &

$h = \frac{8}{12}(12+27)$

$= \frac{2}{3}(39) = 2 \cdot 13 = 26$

& $L = \sqrt{(x+27)^2 + 26^2}$

$= \sqrt{39^2 + 26^2}$

$= \sqrt{(3 \cdot 13)^2 + (2 \cdot 13)^2}$

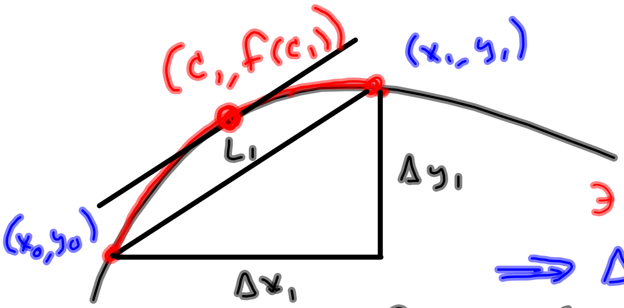
$= \sqrt{3^2 \cdot 13^2 + 2^2 \cdot 13^2}$

$= \sqrt{13^2 (3^2 + 2^2)}$

$= 13\sqrt{13} = L$

≈ 46.87216658

§6.3 Arc Length



Mean Value Theorem says.

$\exists\, c_1 \in (x_0, x_1)$

$\ni f'(c_1) = \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \dfrac{\Delta y_1}{\Delta x_1}$

$\Longrightarrow \Delta y_1 = f'(c_1)\,\Delta x_1$

Arc length from (x_0, y_0) to (x_1, y_1) is approximately the length of the secant line.

$$L_1 = \sqrt{(\Delta x_1)^2 + (\Delta y_1)^2}$$

$$= \sqrt{(\Delta x_1)^2 + (f'(c_1)\Delta x_1)^2}$$

$$= \sqrt{(\Delta x_1)^2 + f'(c_1)^2\,(\Delta x_1)^2}$$

$$= \sqrt{(\Delta x_1)^2\left(1 + f'(c_1)^2\right)}$$

$$= \sqrt{(\Delta x_1)^2}\,\sqrt{1 + f'(c_1)^2}$$

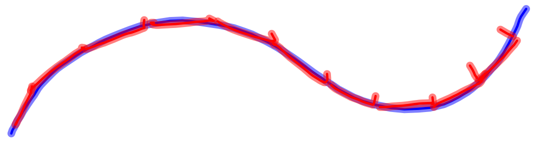
$$= |\Delta x_1|\sqrt{1 + f'(c_1)^2} \qquad \text{Assume } \Delta x_1 \geq 0$$

$$= \sqrt{1 + f'(c_1)^2}\;\Delta x_1$$

$\sqrt{x^2y^2} = |x||y|$

$\cancel{\sqrt{x^2+y^2} = |x|+|y|}$

So do this for a bunch of line segments,



L = length of the curve

$$\approx \sum_{k=1}^{n} \sqrt{1 + \left(f'(c_k)\right)^2}\,\Delta x_k$$

x_{k-1}

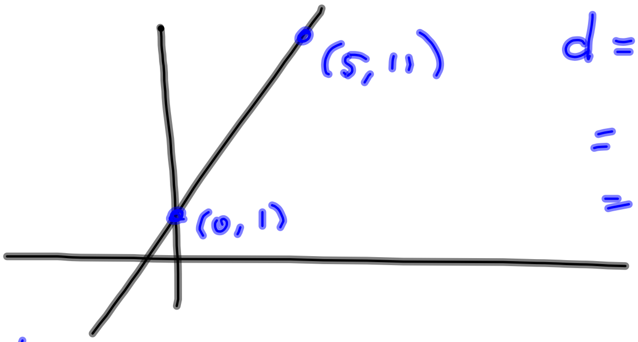
$$\xrightarrow{n \to \text{BIG}} \approx \sum_{k=1}^{n} \sqrt{1 + \left(f'(x_k)\right)^2}\,\Delta x_k$$

$$\xrightarrow{n \to \infty} \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$$

Easy concept. Easy Setup.
Integrals tend to be difficult.

Arc Length for $f(x) = 2x+1$

from $x=0$ to $x=5$



$$d = \sqrt{(5-0)^2 + (11-1)^2}$$
$$= \sqrt{25+100}$$
$$= \sqrt{125}$$
$$= 5\sqrt{5}$$

§6.3 way:

$f'(x) = 2$

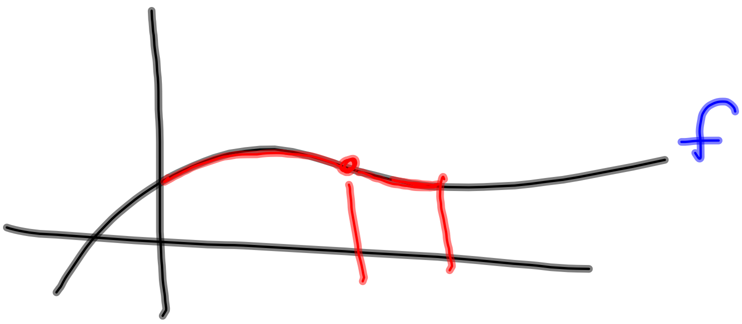
$f'(x)^2 = 2^2 = 4$

$$L = \int_0^5 \sqrt{1+4}\,dx$$
$$= \sqrt{5}\int_0^5 dx$$
$$= \sqrt{5}\,\Big[x\Big]_0^5$$
$$= \sqrt{5}\,[5-0]$$
$$= 5\sqrt{5}$$

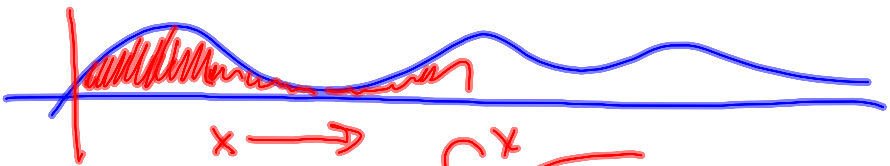
I'm having FTC I thoughts.

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$L(x) = s(x) = \int_a^x \sqrt{1 + (f'(t))^2}\, dt$$



$$\sqrt{1 + (f'(t))^2}$$



$x \longrightarrow$

a So does $\int_a^x \sqrt{\sim}$

$$L(x) = s(x) = \int_a^x \sqrt{1 + (f'(t))^2}\, dt$$

So $\frac{ds}{dx} = \sqrt{1+(f'(x))^2}$

So the differential $ds = \sqrt{1+(f'(x))^2}\, dx$

And so Arc Length L can be written

as $L(x) = \int_a^x ds$ is handy if you have a bunch to do with the same $f(x)$

$f(x) = \sqrt{x}$

Set up:

from 1 to x

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(x)^2 = \frac{1}{4x}$

$$L(x) = \int_1^x \sqrt{1 + \frac{1}{4t}}\, dt$$

Very tough integral with current skill set.

NUMERICAL/ TECHNO.

$$f(x) = y = \int_0^x \tan(t)\,dt \qquad 0 \le x \le \frac{\pi}{6}$$

$$f'(x) = \tan(x)$$

$$f'(x)^2 = \tan^2(x)$$

$$L = \int_0^x \sqrt{1+\tan^2(t)}\,dt = \int_0^x \sqrt{\sec^2(t)}\,dt$$

$$= \int_0^x \sec t\,dt$$ we don't know, yet.

$$= \ln|\sec x + \tan x| \Big]_0^x$$ by tricks the master hasn't yet given (but one time)

When you have several to do, it's nice to find the antiderivative & do this $[F(x) - F(a)]$

plug in different x's as needed.