$$\int_{4-\sqrt{19}}^{4+\sqrt{19}} |x^3 - 10x^2 + 13x + 6|\, dx$$

$$= \int_{4-\sqrt{19}}^{2} (x^3 - 10x^2 + 13x + 6)\, dx + \int_{2}^{4+\sqrt{19}} (-x^3 + 10x^2 - 13x - 6)\, dx$$

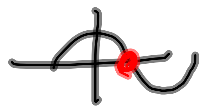
B/c

$$|x^3 - 10x^2 + 13x + 6| = \begin{cases} x^3 - 10x^2 + 13x + 6 & 4-\sqrt{19} \le x \le 2 \\ -x^3 + 10x^2 - 13x - 6 & 2 < x \le 4+\sqrt{19} \end{cases}$$

$$F(2) \underbrace{- F(4-\sqrt{19})}_{\text{same}} + \underbrace{- F(4+\sqrt{19})}_{\text{same}} \underbrace{- (-F(2))}_{\text{Deceptive}}$$

$$= -F(4-\sqrt{19}) - F(4+\sqrt{19}) + 2F(2)$$

If $F = \int (x^3 - 10x^2 + 13x + 6)\, dx$

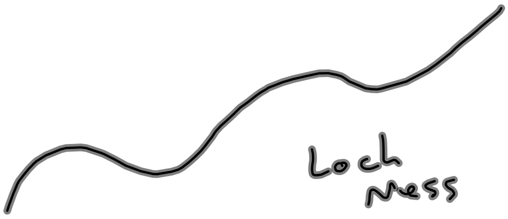


$$\int_0^2 2x\, dx = x^2 \Big]_0^2 = 4 - 0 = 4$$

$$= \int_0^1 2x\, dx + \int_1^2 2x\, dx$$

$$= x^2 \Big]_0^1 + x^2 \Big]_1^2$$

$$= 1 - 0 + 4 - 1$$



Loch Ness $\frac{\sqrt{3}}{2}x + \cos x$

Terraces $\begin{cases} x + \cos x \\ 1 - \sin x = f'(x) = 0 \end{cases}$

$\frac{\sqrt{3}}{2}x + \cos x$ on $[0, 2\pi]$

$f(0) = 1 \qquad (0, 1)$

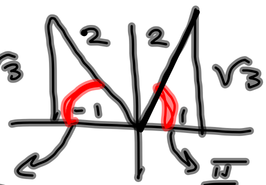
$f(2\pi) = \frac{\sqrt{3}}{2} \cdot 2\pi + 1$

$= \sqrt{3}\pi + 1 \qquad (2\pi, \sqrt{3}\pi + 1)$

$f'(x) = \frac{\sqrt{3}}{2} - \sin x \overset{SET}{=} 0$

$\sin x = \frac{\sqrt{3}}{2}$

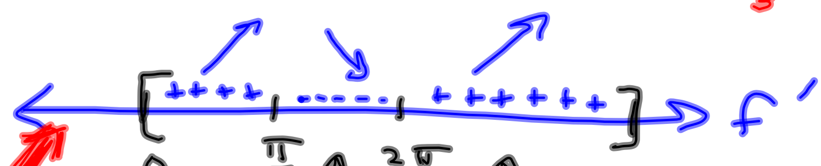
$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$



$\frac{\pi}{3}$

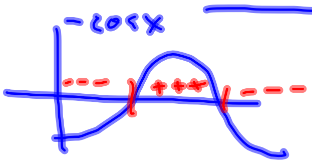
$\pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$



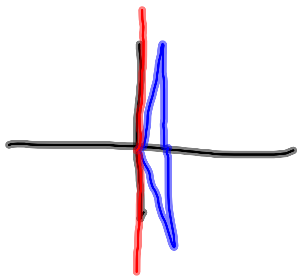
Test: $0, \frac{\pi}{2}, \frac{4\pi}{3}$

$\underline{f''(x) = -\cos x} \overset{SET}{=} 0$



$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$



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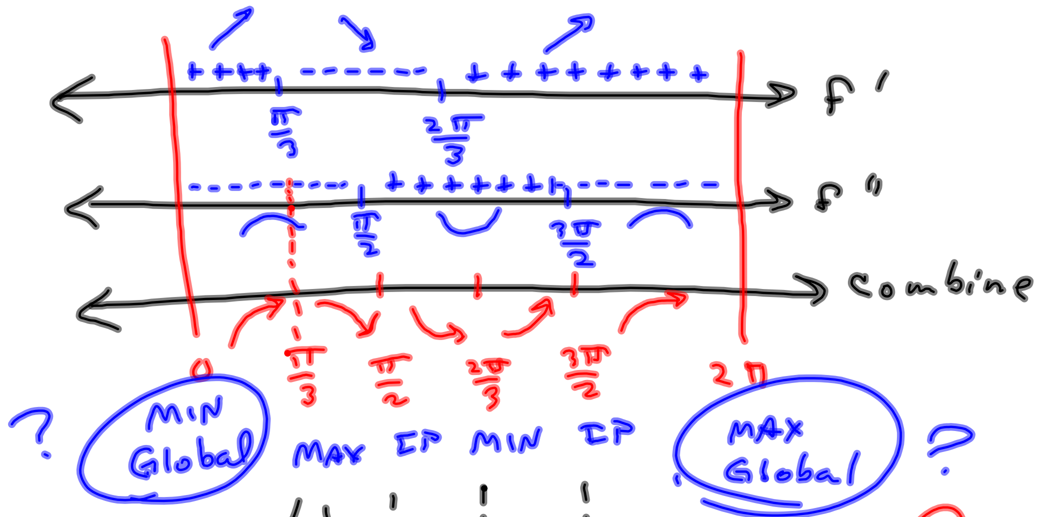
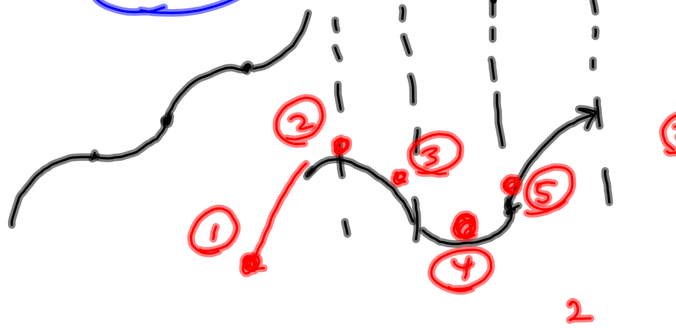




① $f(0) = 1$ ① $(0, 1)$ E.P.

② $f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \cdot \frac{\pi}{3} + \frac{1}{2}$

$= \frac{\sqrt{3}\pi}{6} + \frac{1}{2}$

≈ 1.407

② $\left(\frac{\pi}{3}, 1.407\right)$ MAX

③ $f\left(\frac{\pi}{2}\right) = \frac{\sqrt{3}}{2} \cdot \frac{\pi}{2} = \frac{\sqrt{3}\pi}{4}$

≈ 1.360

③ $\left(\frac{\pi}{2}, 1.360\right)$ I.P.

④ $f\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}\pi}{3} - \frac{1}{2}$

≈ 1.314

④ $\left(\frac{2\pi}{3}, 1.314\right)$ MIN

⑤ $f\left(\frac{3\pi}{2}\right) = \frac{\sqrt{3}}{2} \cdot \frac{3\pi}{2}$

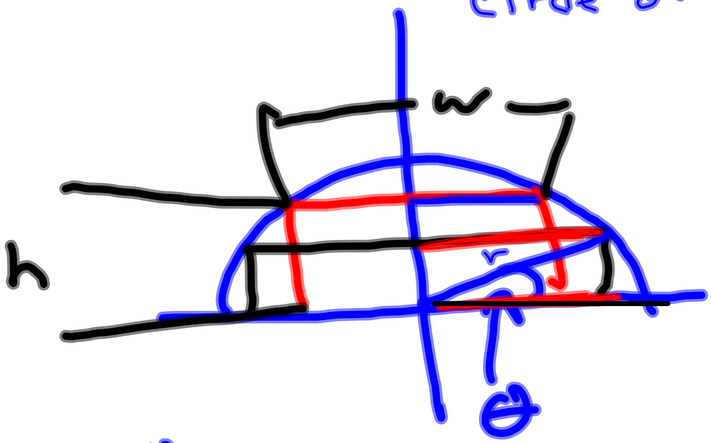
$= \frac{3\sqrt{3}\pi}{4}$

≈ 4.081

⑤ $\left(\frac{3\pi}{2}, 4.081\right)$ IP

Applied Optimization

circle of radius 3



$\frac{h}{r} = \sin\theta$

$h = r\sin\theta$

$= 3\sin\theta$

$\frac{w/2}{r} = \cos\theta$

$w = 6\cos\theta$

$0 \le \theta \le \frac{\pi}{2}$

$\text{Area} = hw = 18\sin\theta\cos\theta$

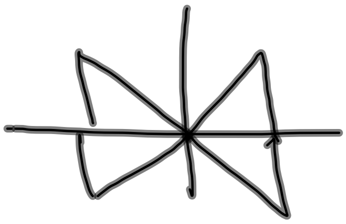
Maximize it:

$$\frac{dA}{d\theta} = 18\left[\cos^2\theta - \sin^2\theta\right] \overset{SET}{=} 0$$

$\cos^2\theta = \sin^2\theta$

$\cos\theta = \pm\sin\theta$

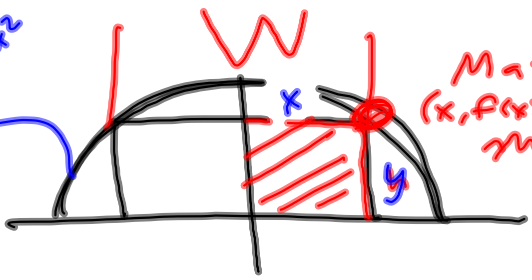
$\theta = \frac{\pi}{4}$



So $w = 6\cos\frac{\pi}{4} = 6\cdot\frac{1}{\sqrt{2}} = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

& $h = 3\sin\frac{\pi}{4} = 3\cdot\frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

$y=\sqrt{9-x^2}$



Max rect by max-ing the little rect

$W=2w$

$$x^2+y^2=3^2$$

$$y^2=9-x^2$$

$$y=\pm\sqrt{9-x^2}$$

$$y=+\sqrt{9-x^2}$$

$$\text{Area}=xy=x\sqrt{9-x^2}=x(9-x^2)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dA}{dx}=(9-x^2)^{\frac{1}{2}}+x\left(\tfrac{1}{2}(9-x^2)^{-\frac{1}{2}}\right)(-2x)$$

$$\frac{\sqrt{9-x^2}}{\sqrt{9-x^2}}\sqrt{9-x^2}-\frac{x^2}{\sqrt{9-x^2}}$$

$$=\frac{9-x^2-x^2}{\sqrt{9-x^2}}\overset{\text{SET}}{=}0$$

$$\Rightarrow 9-2x^2=0$$

$$9=2x^2$$

$$\frac{9}{2}=x^2$$

$$x=\pm\sqrt{\frac{9}{2}}=\pm\frac{3}{\sqrt{2}}$$

$$y=\sqrt{9-\left(\frac{3}{\sqrt{2}}\right)^2}$$

$$=\sqrt{9-\frac{9}{2}}$$

$$=\sqrt{\frac{9}{2}}=\frac{3}{\sqrt{2}}$$

$\boxed{x=\frac{w}{2}}\Rightarrow$

$w=\frac{6}{\sqrt{2}}=\frac{6\sqrt{2}}{2}=\boxed{3\sqrt{2}}$

$y=\boxed{h=\frac{3\sqrt{2}}{2}}$