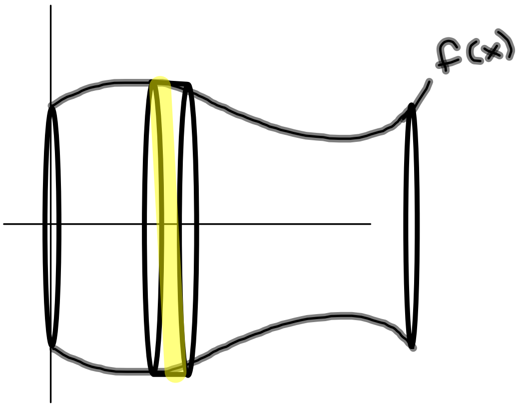
Disk and Washer Method



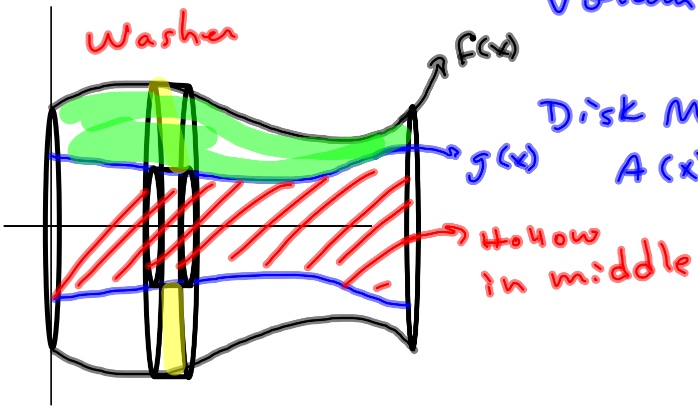
§6.1 Focus on Disk & Washer Method

Read Example 1

$A(x)$ = cross-sectional area @ x.

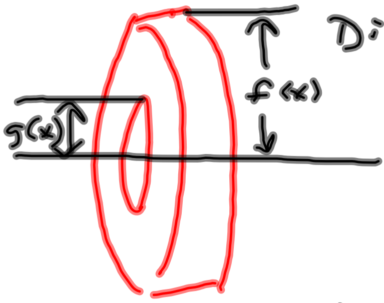
dx = thickness (incremental)

$$\text{Volume} = \int_a^b A(x)\,dx$$



Disk Method:

$$A(x) = \pi f(x)^2$$



Disk: $\pi\int_a^b f(x)^2\,dx$

outer - inner

$= \pi\int_a^b f(x)^2\,dx - \pi\int_a^b g(x)^2\,dx$

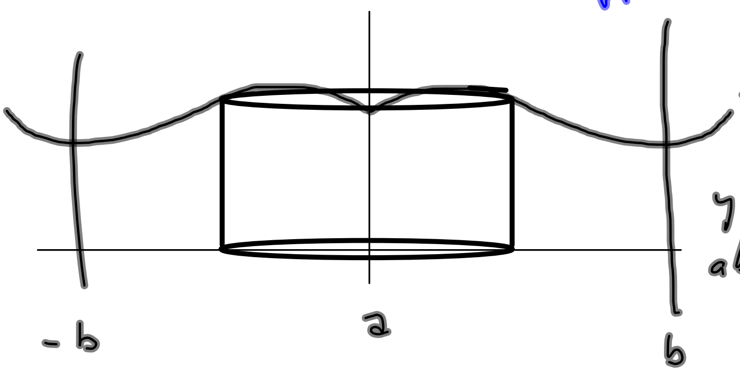
$= \boxed{\pi\int_a^b (f(x)^2 - g(x)^2)\,dx}$ by linearity of $\int$ operator.

$\int$ respects addition & multiplication by a constant

$a\int f + b\int g = \int(af + bg)$

Shell Method

Disk method hard on this one.



y = f(x) rotated about y-axis

volume of cylindrical shell of radius r & thickness Δx & height h is

$$2\pi r h \Delta x$$

$$= 2\pi x f(x) \Delta x = 2\pi x y \Delta x$$

$$\text{volume} = 2\pi \int_a^b x f(x)\, dx$$

Nov 8-10:28 AM

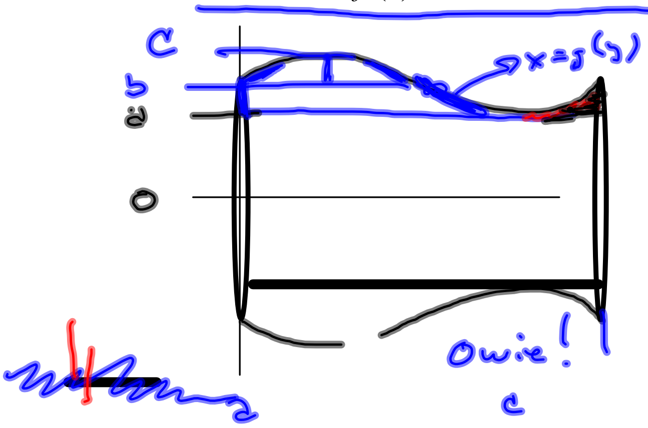
Usually, either Shell or Disk/Washer Method will be preferred.

When f and/or g are 1-to-1, then switching back and forth is fairly easy.

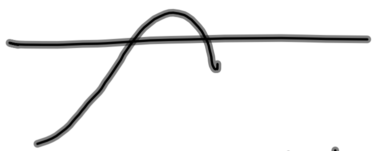
But if a function has local extremes on the region in question, things get dicey very quickly, if you pick a "wrong" method.

This can come up when you have a function of x that is rotated around the x-axis, when you're asked to use the *shell* method. Very difficult. The cylindrical shells might cross over some "dead space" in a trough, for instance.

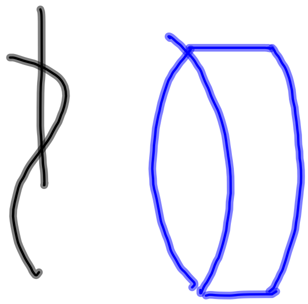
Very tough situation, here. See how the horizontal cylinder leaks outside of the solid? Very difficult to formulate. You'd basically have to find the low point of the function $f(x)$, and then break the volume up into two separate chunks.



y= f(x) not 1-to-1



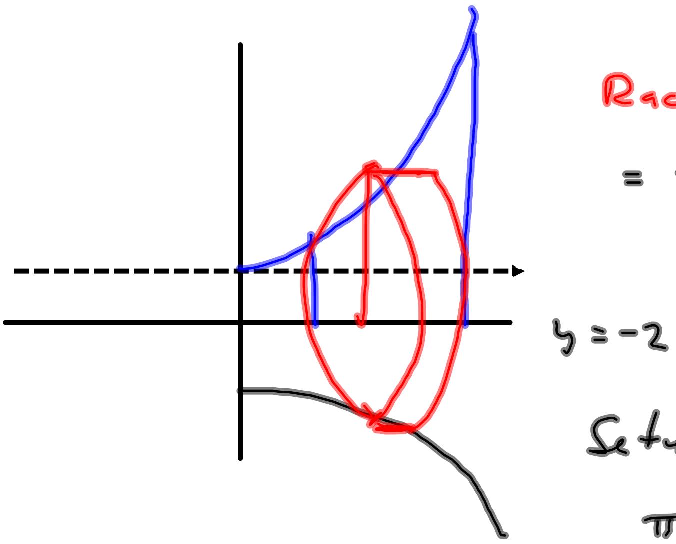
x = g(y) not 1-to-1



Two separate chunks isn't good enough here. The on left side is different, also

Suffice it to say, some methods are better than others on a given problem

$f(x) = x^2$ from $x = 1$ to $x = 5$ about the line $y = -2$



Radius $= f(x) - (-2)$

$= x^2 + 2$

Setup:

$$\pi \int_1^5 (x^2+2)^2 \, dx$$

C4 Practice Test Monday 11/12

C5 Practice Test Monday 11/19

MIDTERM RE-DO Test Tuesday 11/20

C6 2 weeks to cover it ALL

Avoid Dain Bramage in §6.1

Example 1

Arc Length

Surface area of solids of revolution

Neat Apps of integration

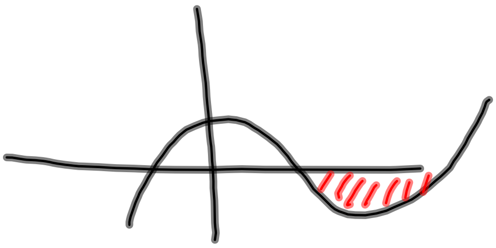
work, force, center of mass.

If $f(x)$ & $g(x)$ are continuous,
then so is $|f(x)-g(x)|$
In practice, you can just do this:

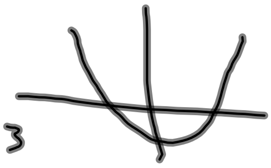
$$\int_a^b |f(x)-g(x)|\,dx = \left| F(x)-G(x) \right|_a^b$$

where F & G are antiderivatives of
f & g, respectively.
There are some technical issues with
actually writing $F(x)-G(x)$

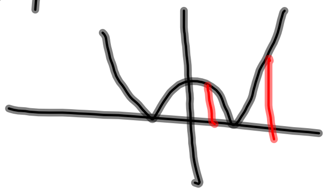
Nov 8-12:49 PM



x^2-4



$$\int_1^3 |x^2-4|\,dx$$



$\frac{x^3}{3}-4x$

$= \frac{1}{3}[x^3-12x]$

$= \frac{1}{3}x[x^2-12]$

$= \frac{1}{3}x[x-2\sqrt{3}][x+2\sqrt{3}]$

$= 0$ when $x=0$, $x=\pm 2\sqrt{3}$

I'm just not convinced that

$$\int_a^b |h| = |H|_a^b$$

You have to be careful, because $H=0$ in different spots than h