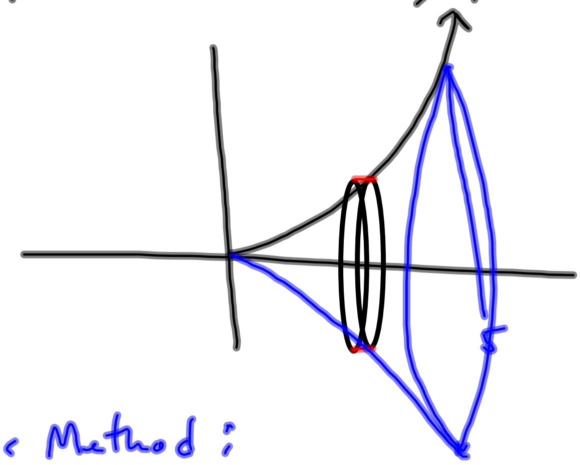
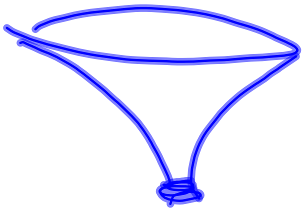
Find the volume of the solid formed when $f(x) = x^2$ is rotated about the x-axis, from $x = 0$ to $x = 5$





Disc Method :

$$\text{volume} \approx \sum_{k=1}^{n} \pi \left(f(x_k)\right)^2 \Delta x$$

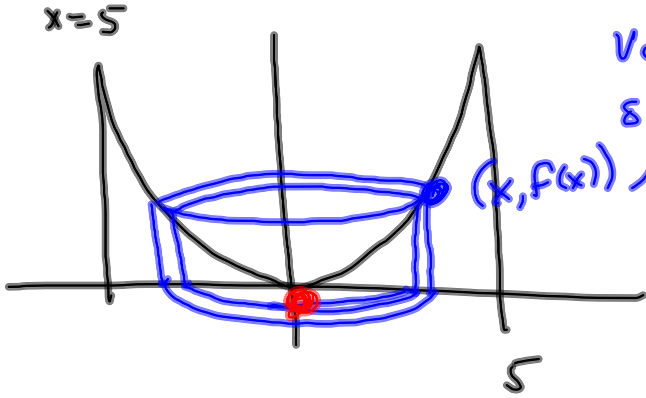
$$\xrightarrow{n \to \infty} \int_0^5 \pi (x^2)^2 dx$$

$$= \pi \int_0^5 x^4 dx = \pi \left[\frac{1}{5} x^5\right]_0^5 = \frac{\pi}{5}\left[5^5\right] = 5^4 \pi$$

$$= 625\pi$$

Shell method

Find the volume when $f(x) = x^2$ is rotated about the y-axis from $x=0$ to $x=5$

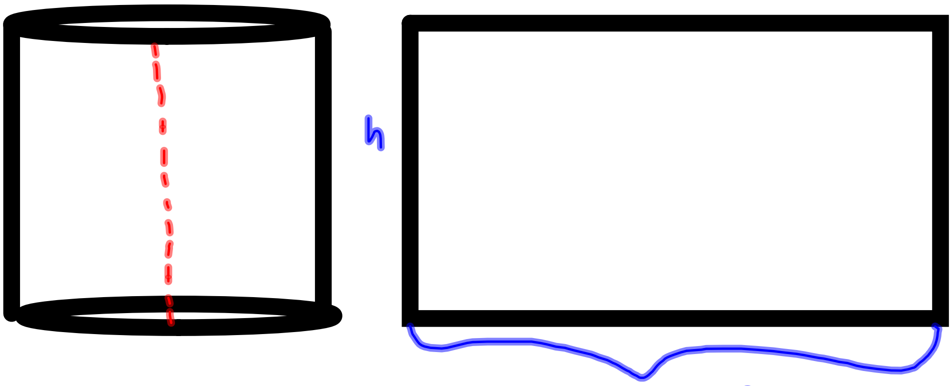


$(x, f(x))$

Volume of cylindrical shell of radius x and height $f(x)$ and thickness Δx

$= 2\pi x f(x) \Delta x$

$f(x) = h$



$2\pi r = 2\pi x$

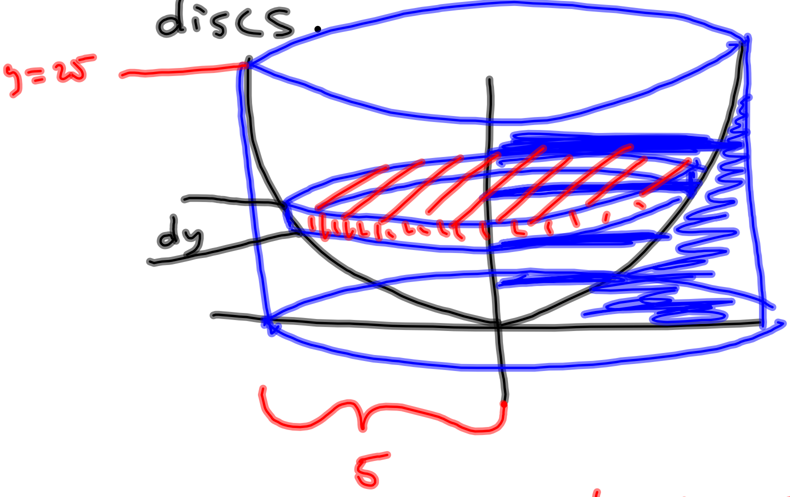
$$\text{Volume} = 2\pi \int_0^5 x \cdot f(x)\,dx$$

$$= 2\pi \int_0^5 x \cdot x^2\,dx = 2\pi \left[\frac{1}{4}x^4\right]_0^5 = \frac{1}{2}\pi\left[625\right]$$

$$= \frac{625\pi}{2}$$

Nov 7-12:17 PM

Same question, only you Must use discs.



Calls for WASHER METHOD Volume between = Outer − Inner.

volume of the outer is $\pi \cdot 5^2 \cdot 25 = \pi r^2 h$
$= 625\pi$

Inner: $\pi \int_0^{25} (g(y))^2 dy = \pi \int_0^{25} (\sqrt{y})^2 dy = \pi \int_0^{25} y \, dy$

($y = x^2 \Longrightarrow x = \pm\sqrt{y}$ use the right half.
$x = \sqrt{y} = g(y)$)

$= \pi \left[\frac{y^2}{2} \right]_0^{25} = \frac{\pi \cdot 625}{2} - \frac{\pi \cdot 0^2}{2}$

$= \frac{625\pi}{2}$

Do the volume of the solid of revolution obtained when $f(x) = x^3$, from $x=1$ to $x=3$ is rotated about...

(1) ... the x-axis

(2) ... " y-axis

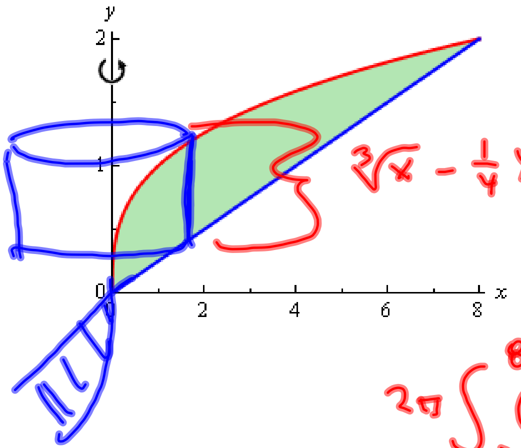
Example 2 Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y-axis.

$\frac{x}{4} = x^{\frac{1}{3}}$ $\qquad$ $\frac{x^3}{64} = x$

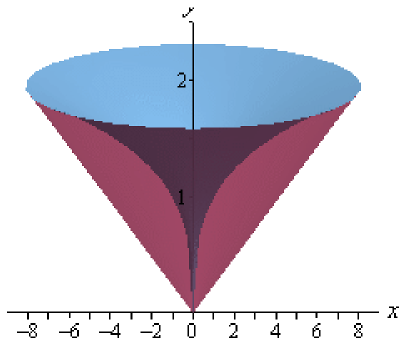
$x^3 - 64x = 0$

$x(x^2-64)=0$

$x=0,\ x=\pm 8$



$\sqrt[3]{x} - \frac{1}{4}x$



$$2\pi\int_0^8 \left(\sqrt[3]{x} - \frac{1}{4}x\right) dx$$