

5.5, 5.6

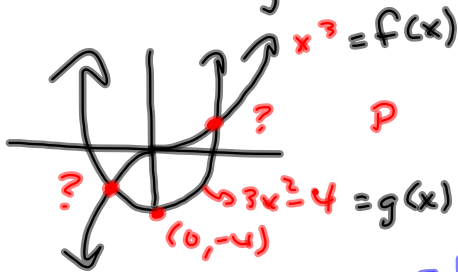
(60)

$$x^3 - y = 0$$

$$3x^2 - y = 4$$

$$y = x^3$$

$$y = 3x^2 - 4$$



$$3x^2 - 4 = x^3$$

$$x^3 - 3x^2 + 4 = 0$$

$$\pm 1, \pm 2, \pm 4$$

$$\begin{array}{r} -1 \mid 1 \quad -3 \quad 0 \quad 4 \\ \quad \quad -1 \quad +4 \quad -4 \\ \hline \quad \quad 1 \quad -4 \quad +4 \quad 0 \end{array}$$

$$x^2 - 4x + 4 = (x-2)^2 = 0$$

$$x = 2$$

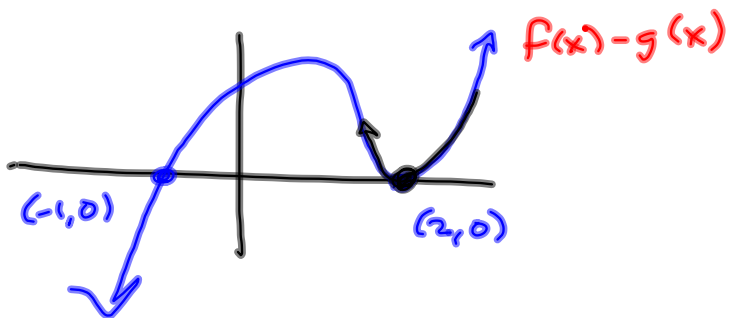
$$f(x) - g(x) = h(x)$$

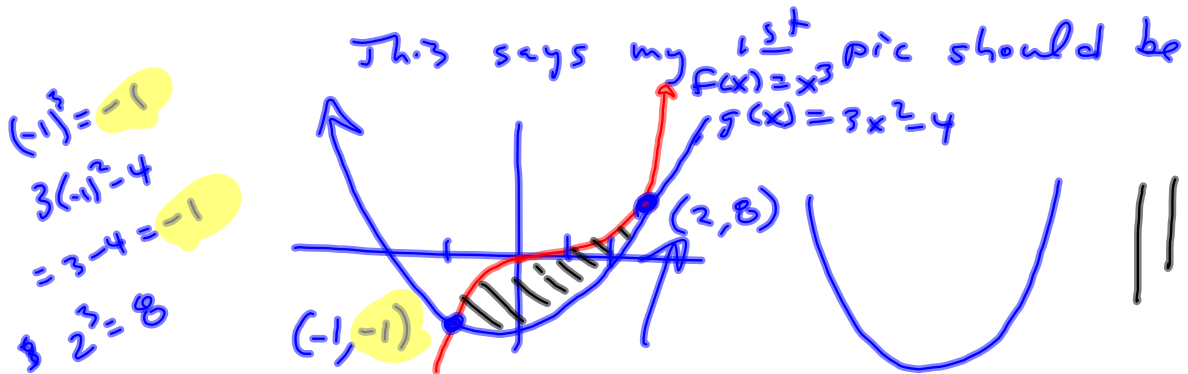
$$= x^3 - 3x^2 + 4$$

$$h(x) = \frac{(x+1)(x-2)^2}{x^3}$$

x^3

...





$$\begin{aligned} (-1)^3 &= -1 \\ 3(-1)^2 - 4 &= 3 - 4 = -1 \\ 2^3 &= 8 \end{aligned}$$



So, Area between is $\int_{-1}^2 (f(x) - g(x)) dx$

$$= \int_{-1}^2 (x^3 - 3x^2 + 4) dx = \left[\frac{1}{4}x^4 - x^3 + 4x \right]_{-1}^2$$

$$= \frac{1}{4}(16) - 8 + 8 - \left(\frac{1}{4}(-1)^4 - (-1)^3 + 4(-1) \right)$$

$$= 4 - \left(\frac{1}{4} + 1 - 4 \right) = 4 - \frac{1}{4} + 3 = 7 - \frac{1}{4} = \frac{28-1}{4} = \frac{27}{4}$$

$$\int_0^{\frac{\pi}{4}} \frac{\sin \theta}{\sin(\cos \theta)} d\theta$$

$$u = \cos \theta$$

$$du = -\sin \theta d\theta$$

$$d\theta = \frac{du}{-\sin \theta}$$

$$\cos(0) = 1$$

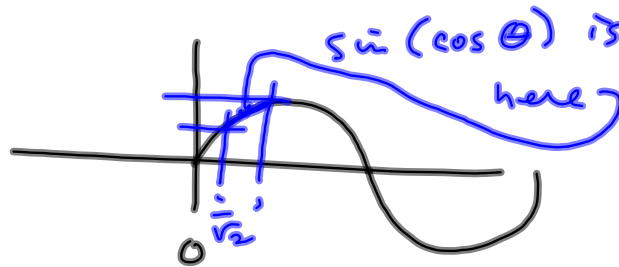
$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}$$

$$= \int_{\frac{1}{\sqrt{2}}}^1 \frac{\cancel{\sin \theta}}{\sin(u)} \cdot \frac{du}{-\cancel{\sin \theta}}$$

on $[0, \frac{\pi}{4}]$, $\cos \theta \in [\frac{1}{\sqrt{2}}, 1]$

$\sin(\cos \theta) \in$

$$= - \int_{\frac{1}{\sqrt{2}}}^1 \frac{du}{\sin u}$$



$$= \int_{\frac{1}{\sqrt{2}}}^1 \csc u du$$

But idiot teacher forgot we can't do $\int \csc u du$, yet.

$$\int (\csc u) \left(-\frac{\csc u + \cot u}{\csc u + \cot u} \right) du$$

$$= - \int \frac{(\csc^2 u + \csc u \cot u)}{\csc u + \cot u} du$$

$$= - \int \frac{\csc^2 u - \csc u \cot u}{\cot u + \csc u} du \quad \text{Let } v = \cot u + \csc u$$

$$= - \int \frac{dv}{v} = -\ln|v| + C$$

$$= -\ln|\csc u + \cot u| + C$$

$$\bigcirc \int \frac{\sqrt{x^3-3}}{\sqrt{x^{11}}} dx$$

$$\frac{x^3-3}{x^{11}} = \frac{x^3}{x^{11}} - \frac{3}{x^{11}} = \frac{1}{x^8} - \frac{3}{x^{11}} *$$

$$\frac{x^3-3}{x^8 \cdot x^3} = \frac{1}{x^8} \left(\frac{x^3-3}{x^3} \right) = \frac{1}{x^8} \left(1 - \frac{3}{x^3} \right)$$

$$\int \frac{1}{\sqrt{x^8}} \sqrt{1 - \frac{3}{x^3}} dx \quad \text{must assume } x > 0$$

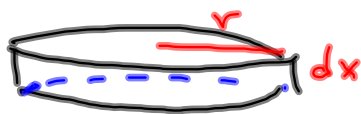
$$= \int x^{-4} \left(1 - 3x^{-3} \right)^{\frac{1}{2}} dx \quad \sqrt{x^8} = |x^4| = x^4$$

$$u = 1 - 3x^{-3}$$

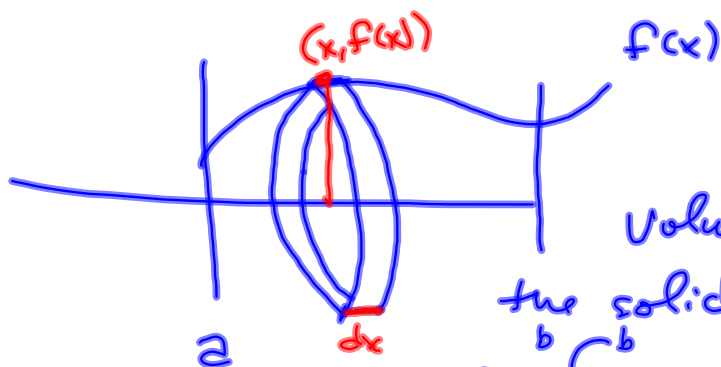
$$du = 9x^{-4} dx$$

$$dx = \frac{du}{9x^{-4}}$$

$$\int \cancel{x^{-4}} (u)^{\frac{1}{2}} \frac{du}{9 \cancel{x^{-4}}} = \frac{1}{9} \int u^{\frac{1}{2}} du$$

Scan
§6.1, 6.2

$$\text{Volume} = \pi r^2 dx$$



Volume of
the solid of revolution
is $\int_a^b \pi (f(x))^2 dx$