Recall last time, we worked on §4.2 #24.
Karla had the best argument, using a wacky angle addition formula

Recall we wanted to show that
$r(\theta) = 2\theta - \cos^2\theta + \sqrt{2}$ had exactly one real zero.

Sketch: use IVT to show $\exists$ @ least one.
Use $r'(\theta)$ to show $\exists$ @ MOST one.
$\Rightarrow \exists$ exactly one.

$\frac{1}{2}\sin(A-B) - \frac{1}{2}\sin(A+B) = \sin A \cos B$
was Karla's thing

$$r'(\theta) = 2 + 2\cos\theta\sin\theta$$
$$= 2 + 2\left[\frac{1}{2}\sin(\theta-\theta) - \frac{1}{2}\sin(\theta+\theta)\right]$$
$$= 2 + \sin(2\theta) > 0, \text{ since } -1 \le \sin(2\theta) \le 1$$

(i.e. $-1+2 \le 2+\sin(2\theta) \le 1+2$
$1 \le \cdots \le 3$
i.e. $2+\sin(2\theta) \ge 1 > 0$)

My way: $2\sin\theta\cos\theta = \sin(2\theta)$ is slicker.

When I worked it out B4 class, I was too dumb to remember (or look) the identity.

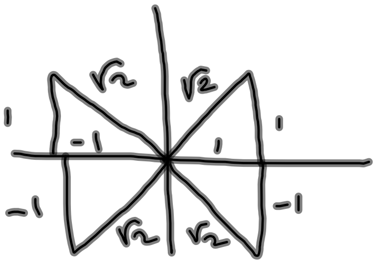
$r'(\theta) = 2 + 2\cos\theta\sin\theta$

& want to argue $\boxed{r'(\theta) > 0}$

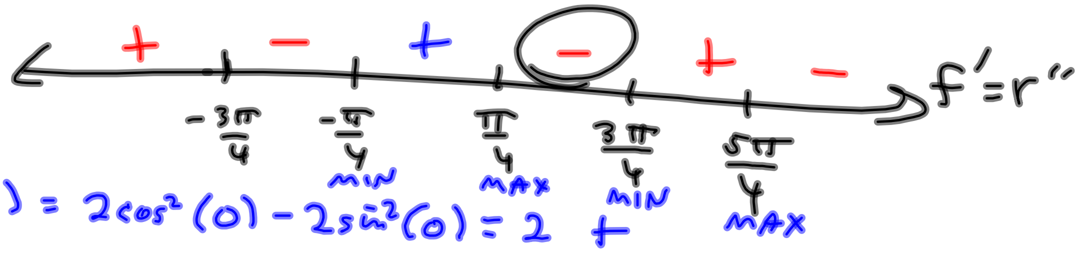
Find max/min values of

$f(\theta) = r'(\theta)$

$$f'(\theta) = 2(-\sin\theta)\sin\theta + 2(\cos\theta)\cos\theta$$
$$= -2\sin^2\theta + 2\cos^2\theta \overset{\text{SET}}{=} 0$$
$$\sin^2\theta = \cos^2\theta$$
$$\sqrt{\sin^2\theta} = \sqrt{\cos^2\theta}$$
$$|\sin\theta| = |\cos\theta|$$
$$\sin\theta = \pm\cos\theta$$



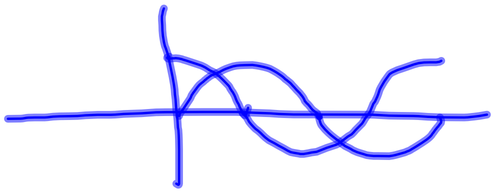
$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} + 2n\pi \quad n\in\mathbb{Z}$$
$$\theta = \frac{(2n+1)}{4}\pi, \quad n\in\mathbb{Z}$$
$$\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \ldots$$



Test:

$$f'(0) = 2\cos^2(0) - 2\sin^2(0) = 2 \quad +$$

$$2\underline{(\cos\theta - \sin\theta)}(\cos\theta + \sin\theta) = f'(\theta)$$



$$f'\left(\tfrac{\pi}{2}\right) = 2\cos^2\left(\tfrac{\pi}{2}\right)$$
$$-2\sin^2\left(\tfrac{\pi}{2}\right) = -2 \quad -$$

$$f\left(\tfrac{\pi}{4}\right) =$$

$$f\left(-\tfrac{\pi}{4}\right) = 2 + 2\cos\left(-\tfrac{\pi}{4}\right)\sin\left(-\tfrac{\pi}{4}\right)$$
$$= 2 + 2\cdot\frac{1}{\sqrt{2}}\cdot\left(-\frac{1}{\sqrt{2}}\right) = 2 - 2\cdot\frac{1}{2} = 1 > 0$$

DONE.

5.5, 5.6 :

① $\int \frac{10\sqrt{x}}{(x^{3/2}+1)^2}$

② (a) 5.5 #40

(b) 5.5 #42

③ See yesterday's notes.

$$\int_2^7 x^2\,dx \qquad \Delta x = \frac{7-2}{n} = \frac{5}{n}$$

$$x_k = 2 + \frac{5k}{n}$$

$$\sum_{k=1}^{n} f(x_k)\,\Delta x = \sum_{k=1}^{n}\left(2+\frac{5k}{n}\right)^2\cdot\frac{5}{n}$$

$$=\frac{5}{n}\sum_{k=1}^{n}\left(4+\frac{20k}{n}+\frac{25k^2}{n^2}\right)$$

$$\begin{array}{r} {}^{6}49 \\ \times\ 7 \\ \hline 343 \end{array}$$

$$\int_2^7 x^2\,dx = \frac{1}{3}\left[x^3\right]_2^7 = \frac{1}{3}\left[7^3-2^3\right] = \frac{1}{3}\left[343-8\right] = \frac{1}{3}\left[335\right]$$

$$=\frac{5}{n}\left[4\sum_{k=1}^{n}1+\frac{20}{n}\sum_{k=1}^{n}k+\frac{25}{n^2}\sum_{k=1}^{n}k^2\right]$$

$$=\frac{5}{n}\left[4n+\frac{20}{n}\cdot\frac{n^2+\cdots}{2}+\frac{25}{n^2}\cdot\frac{n^3+\cdots}{3}\right]$$

$$=20+\frac{100n^2}{2n^2}+\cdots+\frac{125n^3}{3n^3}+\cdots$$

$$\xrightarrow{n\to\infty} 20+50+\frac{125}{3} = \frac{210+125}{3} = \frac{335}{3}$$

$$\sum_{k=1}^{n}k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+\cdots}{6} = \frac{n^3+\cdots}{3}$$

$$\sum_{k=1}^{n}k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \boxed{\frac{n^4+\cdots}{4}}$$

$$\sum_{k=1}^{n}k^{17} = \boxed{\frac{n^{18}+\cdots}{18}}$$

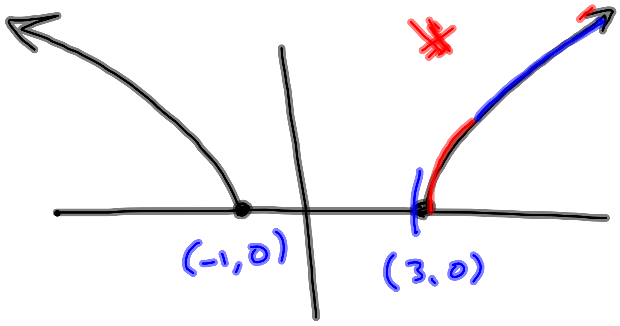
$$\int x^{17}\,dx = \frac{x^{18}}{18}+C$$

#50

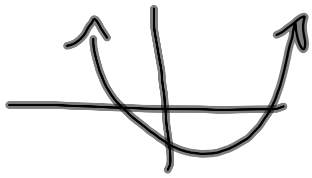
$f(x) = \sqrt{x^2-2x-3}$

$= \sqrt{(x-3)(x+1)}$

Find all extremes
Book says on $[3,\infty)$
Let's do it in general.



$\emptyset$ $\emptyset$



x^2-2x-3

$$f'(x) = \frac{1}{2}(x^2-2x-3)^{-\frac{1}{2}}(2x-2) = \frac{x-1}{\sqrt{x^2-2x-3}}$$

$$= \boxed{\frac{x-1}{\sqrt{(x+1)(x-3)}}}$$

Critical values:

$x = +3, -1$

$x = 1 \notin \mathcal{D}(f)$

NOT a candidate for max/min, but it DOES play a role in the sign pattern for $f'(x)$.

Irrelevant → Full of it.

We can't worry about $x=1$, dummy, because f' isn't defined between $x=-1$ & $x=3$ same as f isn't.

Min @ $(-1,0)$ & $(3,0)$

Absolute Mins.

Book wanted it on $[3,\infty)$

So $(3,0)$ is local min AND absolute min.