§4.2 #24

$r(\theta) = 2\theta - \cos^2\theta + \sqrt{2}$

Show $r(\theta)$ has exactly one real zero.

$r(-100) = -200 - \cos^2(-100) + \sqrt{2} < 0$

$r(100) = 200 - \cos^2(-100) + \sqrt{2} > 0$

$\Rightarrow$ at <u>least</u> one real zero.

For at <u>MOST</u> one real zero, we need $r'(\theta) > 0$

$r'(\theta) = 2 - 2\cos\theta(-\sin\theta)$

$= 2 + 2\cos\theta\sin\theta$

$= 2 + \sin(2\theta)$

$\sin(2\theta) = 2\sin\theta\cos\theta$ is an identity from MAT 122.

So $1 \le r'(\theta) \le 3 \Rightarrow$

$r'(\theta) > 0$, so there's at <u>most</u> one real zero.

To "see" it.

$\sin(2\theta) \le 1 \Rightarrow$

$\sin(2\theta) + 2 \le 3$

$\sin(2\theta) \ge -1 \Rightarrow$

$\sin(2\theta) + 2 \ge 1$

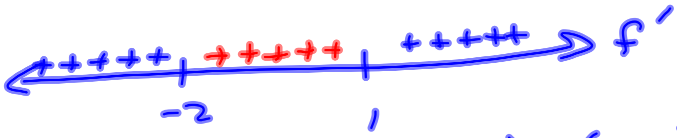
$1 \le r'(\theta) \le 3$

$r'(\theta) > 0$

4.3 #4

$f'(x) = (x-1)^2(x+2)^2$

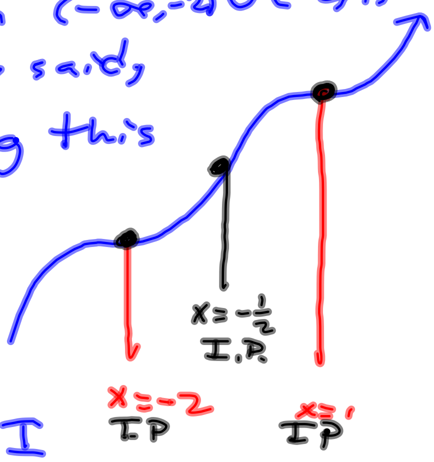
where's it increasing/decreasing?



Increasing on $(-\infty,-2) \cup (-2,1) \cup (1,\infty)$

is what some said,

f is doing this



Increasing on I

means $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ pg 6

So, even though $f'(-2) = 0$,

$x < -2 \Rightarrow f(x) < f(-2)$

and $x > -2 \Rightarrow f(x) > f(-2)$

So increasing @ $x = -2$.

$f''(x) = 2(x-1)(x+2)^2 + 2(x+2)(x-1)^2$

$= 2(x-1)(x+2)\left[x+2+x-1\right]$ $\quad 2x+1 = 0$, $x = -\frac{1}{2}$

$= 0$ @ $x = -2, x = 1$ and $x = -\frac{1}{2}$

$f''(x) = 2(x-1)(x+2)(2x+1)$

So, increasing on $(-\infty, \infty)$

Don't throw out $x = 1, x = -2$.

§5.5, 5.6 WS

#2 should be #40, §5.5

I blurred #40 & #42 together into something IMPOSSIBLE. (And, yes, Josh, I left out the durn "dx".)

So, #2
- #2a 5.5 #40
- #2b 5.5 #42

Do your work on separate paper.

Turn it in like regular homework.

One like #3 on the handout.

Find the area bounded by

$f(x) = x^3 - x^2 - 3x - 1$ & $g(x) = 2x^2 - 2$

(1) Graph separately on same axes

(2) Graph $f(x) - g(x)$ and find a, b so that

$$\text{Area} = \int_a^b |f(x) - g(x)|\,dx.$$

Big Red 1 Lee Marvin

$f(x) = 1x^3 - x^2 - 3x - 1$

$\frac{p}{q} : \pm \frac{1}{1}$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -3 & -1 \\ & & 1 & 0 & -3 \\ \hline & 1 & 0 & -3 & -4 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -3 & -1 \\ & & -1 & 2 & 1 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$f(x) = (x+1)(x^2 - 2x - 1)$

$x^2 - 2x - 1 = 0$

$f(a) = 0 \Longleftrightarrow x - a$ is a factor.

$x^2 - 2x = 1$

So,

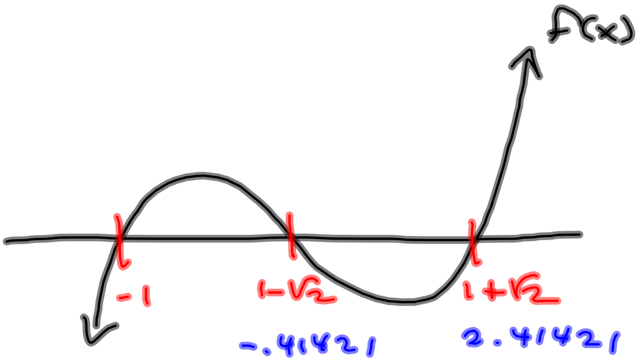
$x^2 - 2x + 1^2 = 1 + 1^2$

$f(x) = (x+1)(x - (1+\sqrt{2}))(x - (1-\sqrt{2}))$

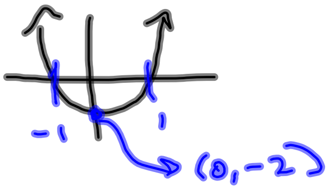
$(x-1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

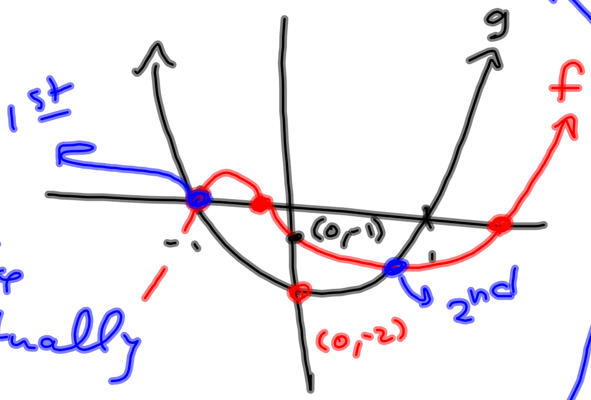
$x = 1 \pm \sqrt{2}$



$g(x) = 2x^2 - 2$



Combine:



$1 - \sqrt{2} \approx -.41421$

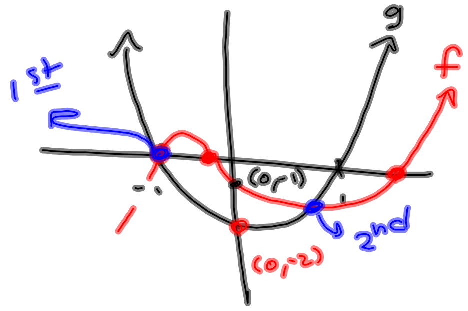
This is an incomplete picture. f(x) eventually is taller than g(x)

$f = x^3 +$ smaller

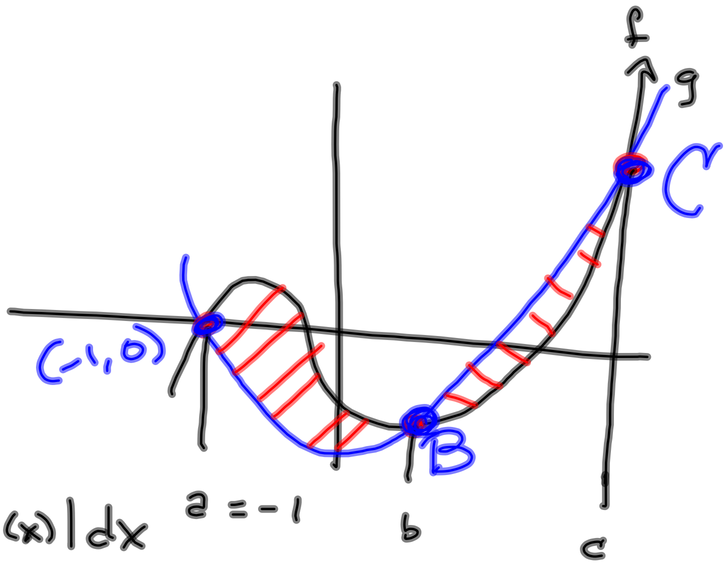
$g = 2x^2 + \;..$

Need to find the 3rd crosser:

combine :



Should be :



$$\text{Area} = \int_a^c |f(x) - g(x)|\,dx$$

$$= \int_a^b (f(x) - g(x))\,dx + \int_b^c (g(x) - f(x))\,dx$$

we need to find a, b, c.

" " " " where $f(x) = g(x)$

" " " " " $f(x) - g(x) = 0$

$f(x) - g(x) = x^3 - x^2 - 3x - 1 - (2x^2 - 2)$

$= x^3 - 3x^2 - 3x + 1$

Split off a factor of $x+1$ (since $x=-1$ is a zero, by previous work.)

$$\begin{array}{r|rrrr} -1 & 1 & -3 & -3 & 1 \\ & & -1 & 4 & -1 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$f(x) - g(x) = (x+1)(x^2 - 4x + 1)$

$x^2 - 4x + 1 = 0$

$x^2 - 4x = -1$

$x^2 - 4x + 2^2 = -1 + 4$

$(x-2)^2 = 3$

$x - 2 = \pm\sqrt{3}$

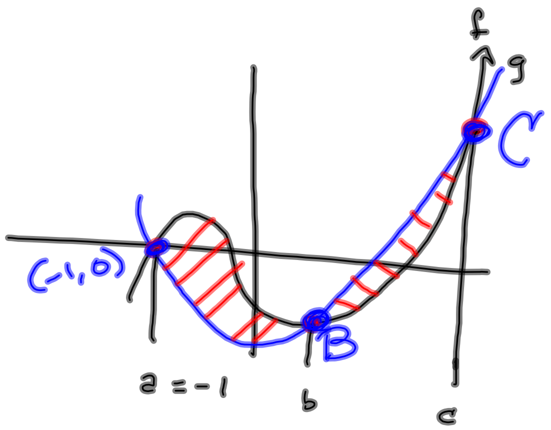
$x = 2 \pm \sqrt{3}$ → $b = 2 - \sqrt{3}$, $c = 2 + \sqrt{3}$

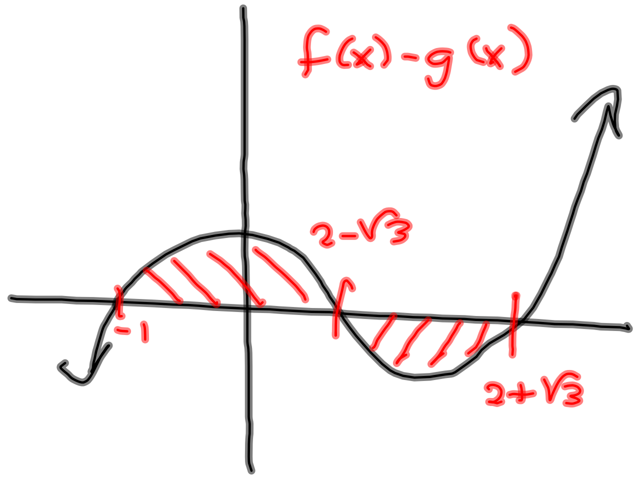
Now, to complete the 1^{st} graph, you need $g(b) = ?$ & $g(c) = ?$

$f(b) = \text{same}$ $\quad f(c) = \text{same}$

$f(2+\sqrt{3}) = 8\sqrt{3} + 12$ $\quad C = (2+\sqrt{3},\ 8\sqrt{3}+12)$

$f(2-\sqrt{3}) = -8\sqrt{3} + 12$ $\quad B = (2-\sqrt{3},\ -8\sqrt{3}+12)$





$2-\sqrt{3} \approx 2-1.732 > 0$

so B is to the right of y-axis ✓

Area

$$= \int_{-1}^{2-\sqrt{3}} (f(x)-g(x))\,dx$$

$$+ \int_{2-\sqrt{3}}^{2+\sqrt{3}} (g(x)-f(x))\,dx$$