FINAL	30
MID	30
HOME	20
PT	20

§ 5.5 #15

$$\int \csc^2(2\theta)\cot(2\theta)\,d\theta$$

$$\left(u = \cot(2\theta) \implies \frac{du}{d\theta} = -2\csc^2(2\theta) \implies du = -2\csc^2(2\theta)\,d\theta \right)$$

$$= -\frac{1}{2}\int \cot(2\theta)\left(-2\csc^2(2\theta)\right)d\theta$$

Q

$$= -\frac{1}{2}\int u\,du = -\frac{1}{2}\frac{u^2}{2} + C = -\frac{1}{4}u^2 + C =$$

$$= \boxed{-\frac{1}{4}\cot^2(2\theta) + C}$$

$u = \csc(2\theta)$

$du = -2\csc(2\theta)\cot(2\theta)$

$$\int \csc^2(2\theta)\cot(2\theta)\,d\theta = -\frac{1}{2}\int \underbrace{\csc(2\theta)}_{u}\underbrace{\left(-2\csc(2\theta)\cot(2\theta)\right)d\theta}_{du}$$

$$= -\frac{1}{2}\frac{u^2}{2} = \boxed{-\frac{1}{4}\csc^2(2\theta) + C}$$

$$-\frac{1}{4}\cot^2(2\theta) + C = -\frac{1}{4}\left(\csc^2(2\theta) - 1\right) + C$$

$$= -\frac{1}{4}\csc^2(2\theta) + \frac{1}{4} + C \to \hat{C}$$

"The $\frac{1}{4}$ is absorbed by the constant."

So $\underbrace{-\frac{1}{4}\csc^2(2\theta) + C = -\frac{1}{4}\cot^2(2\theta) + \hat{C}}_{\text{They differ by a constant.}}$

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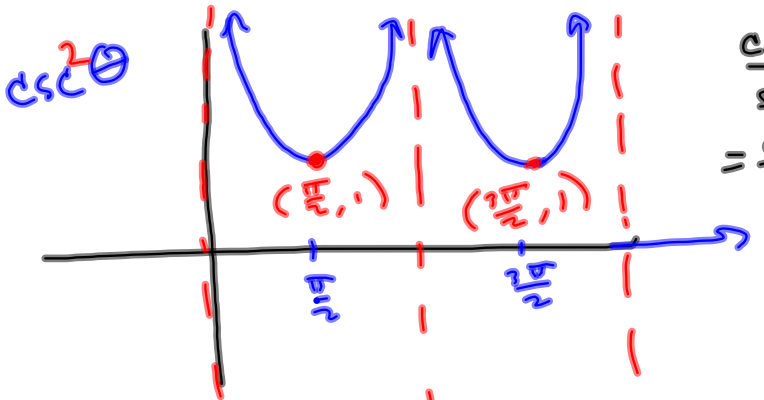
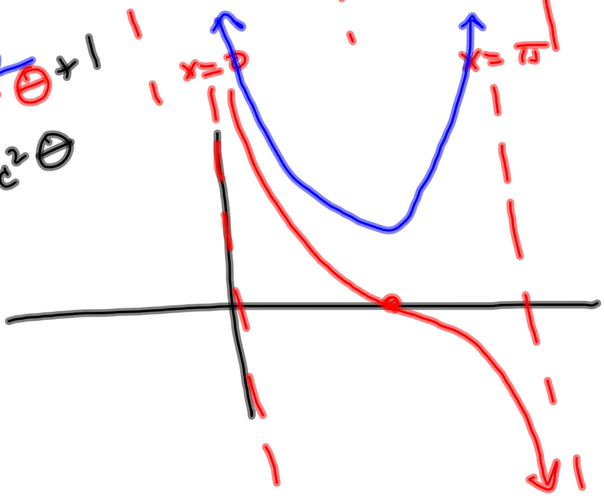
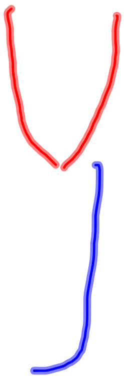


$$\frac{\cos^2\theta}{\sin^2\theta} + 1$$
$$= \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}$$
$$= \frac{1}{\sin^2\theta} = \csc^2\theta$$



$\cot^2\theta + 1$
$= \csc^2\theta$



(13) $\int \sqrt{x}\,\sin^2\left(x^{\frac{3}{2}}-1\right)dx$

$$\left(\begin{array}{l} \text{Let } u = x^{\frac{3}{2}}-1 \longrightarrow \\ \quad du = \frac{3}{2}x^{\frac{1}{2}}dx \end{array}\right)$$

$1-\sin^2(\theta) = \cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$= \frac{2}{3}\int \sin^2\left(x^{\frac{3}{2}}-1\right)\cdot\frac{3}{2}x^{\frac{1}{2}}dx$

$= \frac{2}{3}\int \sin^2 u\, du$

$\sin^2\theta = 1-\cos^2\theta$

$= 1 - \frac{1+\cos(2\theta)}{2}$

$= \frac{2-1-\cos(2\theta)}{2}$

$= \frac{1-\cos(2\theta)}{2}$

$= \frac{2}{3}\int \frac{1-\cos(2x)}{2}\,dx$

$= \frac{2}{6}\int\left(1-\cos(2x)\right)dx$

$= \frac{1}{3}\int 1\,dx - \frac{1}{3}\cdot\frac{1}{2}\int 2\cos(2)\,d$

$v = 2u$
$dv = 2du$

$= \frac{1}{3}x - \frac{1}{6}\int \cos v\,dv$

$= \frac{1}{3}u - \frac{1}{6}\sin v \quad = \frac{1}{3}$

$= \frac{1}{3}\left(x^{\frac{3}{2}}-1\right) - \frac{1}{6}\sin(2x) + C$

$$\int \sin^2(x^{\frac{3}{2}}-1)\sqrt{x}\,dx$$

$u = x^{\frac{3}{2}} - 1$

$du = \frac{3}{2}x^{\frac{1}{2}}dx$

$$= \frac{2}{3}\int \sin^2(x^{\frac{3}{2}}-1)\,\frac{3}{2}x^{\frac{1}{2}}dx$$

$$= \frac{2}{3}\int \sin^2 u\, du$$

$$= \frac{2}{3}\int \frac{1-\cos(2u)}{2}\,du$$

$\int(a-b) = \int a - \int b$

$\frac{1}{2}2du$

$$= \frac{1}{3}\int du - \frac{1}{3}\int \cos(2u)\,du$$

$\left(\begin{array}{l} v = 2u \\ dv = 2du \end{array}\right) = \cancel{\frac{1}{3}\cdot\frac{1}{2}\int \cos(2u)\,2du}$

$= \frac{1}{3}u - \frac{1}{6}\int \cos v\, dv$

so, $du = \frac{dv}{2}$

$$= \frac{1}{3}\int \cos(v)\cdot\frac{dv}{2}$$

$$= \frac{1}{6}\int \cos(v)\,dv$$

$$= \frac{1}{3}(x^{\frac{3}{2}}-1) - \frac{1}{6}\sin v + C$$

$$= \frac{1}{3}(x^{\frac{3}{2}}-1) - \frac{1}{6}\sin(2u) + C$$

$$= \frac{1}{3}(x^{\frac{3}{2}}-1) - \frac{1}{6}\sin\left(2(x^{\frac{3}{2}}-1)\right) + C$$

$$\int (2x-5)\sin(x^2-5x)\,dx = \int (2x-5)\,\sin(u)\,\frac{du}{2x-5}$$

$u = x^2-5x$

$du = (2x-5)\,dx$

$$= \int \sin(u)\,du$$

$dx = \dfrac{du}{2x-5}$

x^2

(14) $\displaystyle\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx$

$u = \frac{1}{x} = x^{-1}$

$du = -\frac{1}{x^2}\,dx$

$dx = -x^2\,du$

$$= \int \frac{1}{x^2}\cos^2(u)\,(-x^2)\,du$$

$$= -\int \cos^2(u)\,du = -\int \frac{1+\cos(2u)}{2}\,du$$

$$= -\int \frac{1}{2}\,du - \int \frac{1}{2}\cos(2u)\,du$$

$v = 2u$

$dv = 2\,du$

$du = \frac{dv}{2}$

$$= -\frac{1}{2}u - \frac{1}{2}\int \cos(v)\,\frac{dv}{2}$$

$$= -\frac{1}{2}\cdot\frac{1}{x} - \frac{1}{4}\sin(v) + C$$

$$= -\frac{1}{2x} - \frac{1}{4}\sin(2u) + C$$

$$= -\frac{1}{2x} - \frac{1}{4}\sin\left(2\cdot\frac{1}{x}\right) + C$$