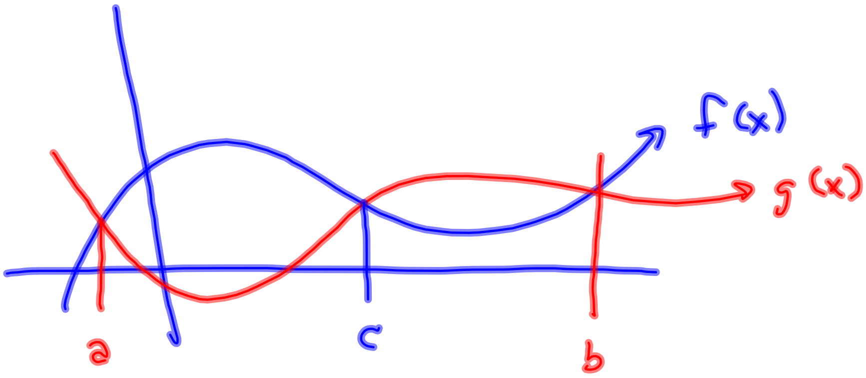
§5.6 Definition of Area Between two curves f & g, on [a,b] is

$$\int_a^b |f(x)-g(x)|\,dx$$

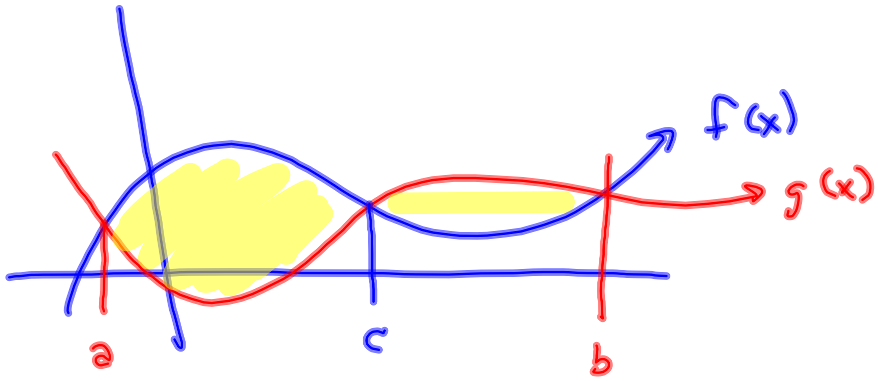


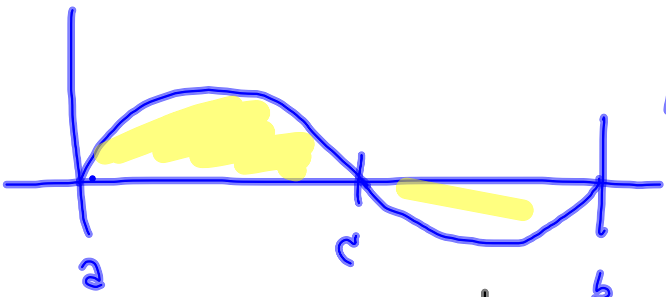
$$\int_a^b |f(x)-g(x)|\,dx = \int_a^c \big(f(x)-g(x)\big)\,dx + \int_c^b \big(g(x)-f(x)\big)\,dx$$

Your job is to find where they cross.
Or, just graph $f(x)-g(x)$ & find its x-intercepts.

$$f(x) = g(x)$$
$$\Longleftrightarrow$$
$$f(x)-g(x) = 0$$

Then analyze where $h(x) = f(x)-g(x)$ is +/−.





$h(x) = f(x) - g(x)$

$$\text{Area} = \int_a^c h(x)\,dx - \int_c^b h(x)\,dx$$

Oct 29-12:14 PM

$$\boxed{(x+2)(x-1)(x+3)}$$

$$= (x+2)(x^2+2x-3)$$

$$= x^3+2x^2-3x$$
$$\quad +2x^2+4x-6$$
$$\overline{x^3+4x^2+x-6}$$

$$= x^3+5x^2+3x-5$$
$$\quad -(x^2+2x+1)$$

Find the area between
$f(x) = x^3+5x^2+3x-5$
and $g(x) = x^2+2x+1$

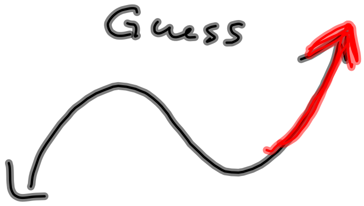
$h(x) = f(x)-g(x) = x^3+4x^2+x-6$

Oct 29-12:24 PM

$h(x) = f(x) - g(x) = x^3 + 4x^2 + x - 6$

So $\int_a^b |f-g| = \int_a^b |h(x)|\,dx$

Guess



$\boxed{x^3 + 4x^2 + x - 6}$

Descartes says: 1 positive zero

$h(-x) = -x^3 + 4x^2 - x - 6$

2 or zero negative zeros.

Rational Zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

Rational Zeros Theorem: Divide by $x-1$

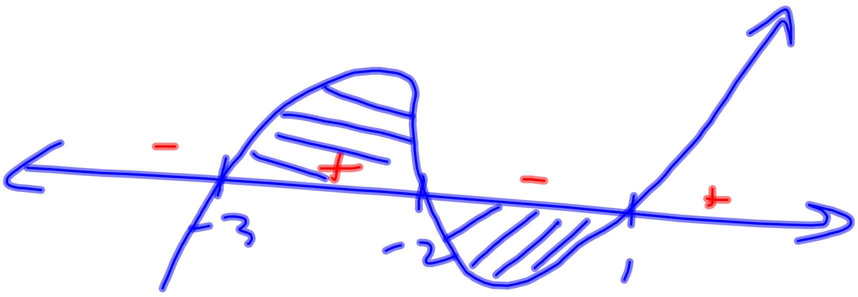
Synthetic Division.

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 1 & -6 \\ & & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

So, $h(x) = (x-1)\boxed{(x^2+5x+6)}$

$= (x-1)(x+3)(x+2)$

Oct 29-12:28 PM



$$\int_{-3}^{-2} h(x) + \int_{-2}^{1} -h(x)$$

Graph this and graph $f(x)$, $g(x)$

$12x^5 - 17x + 35$

$\frac{p}{q}$:

$\pm 1, \pm 5, \pm 7, \pm 35$

$\pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{1}{12}$

$\pm \frac{5}{2}, \pm \frac{5}{3}, \pm \frac{5}{6}, \pm \frac{5}{12}$

$\pm \frac{7}{2}, \pm \frac{7}{3}, \pm \frac{7}{6}, \pm \frac{7}{12}$

$\pm \frac{35}{2}, \pm \frac{35}{3}, \pm \frac{35}{6}, \pm \frac{35}{12}$

Two ways to do definite integrals that involve substitution method.

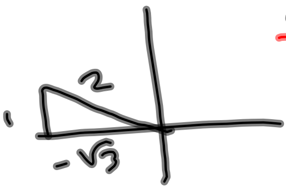
T7 way:

$$\int_a^b f(g(x))\underbrace{g'(x)\,dx}_{du} = \int_{g(a)}^{g(b)} f(u)\,du$$

$$\int_0^{\frac{\pi}{6}} \cos(5x)\,dx = \frac{1}{5}\int_0^{\frac{\pi}{6}} \cos(5x)\cdot 5\,dx = \frac{1}{5}\int_0^{\frac{5\pi}{6}} \cos(u)\,du$$

$u = 5x$

$du = 5\,dx$



$x = 0$ $u = 5x = 0$

$x = \frac{\pi}{6}$ $u = 5\cdot\frac{\pi}{6} = \frac{5\pi}{6}$

$$= \frac{1}{5}\Big[\sin(u)\Big]_0^{\frac{5\pi}{6}} = \frac{1}{5}\left[\sin\left(\frac{5\pi}{6}\right) - \sin(0)\right]$$

$$= \frac{1}{5}\left[\frac{1}{2} - 0\right] = \frac{1}{10}$$

METHOD 2: use u-substitution to find the antiderivative & then apply FTC II

$$\int_0^{\frac{\pi}{6}} \cos(5x)\,dx = \frac{1}{5}\int_0^{\frac{\pi}{6}} \cos(5x)\cdot 5dx$$

$u = 5x$

$du = 5dx$

$$= \frac{1}{5}\Big[\sin(5x)\Big]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{5}\left[\sin\left(\frac{5\pi}{6}\right) - \sin(0)\right] = \dots = \frac{1}{10}$$

Oct 29-12:44 PM

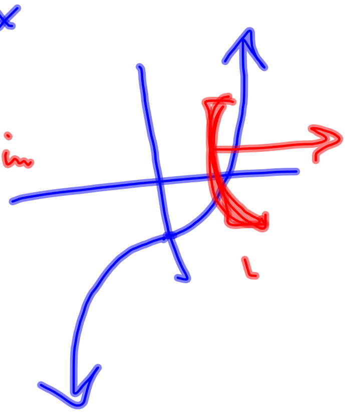
$$\int \sqrt{\frac{x^4}{x^3-1}}\,dx$$

Some concern about $x \geq 0$ & the $x^3-1 \in \sqrt{\quad}$

$u = \dfrac{x^4}{x^3-1} \Longrightarrow du =$ OMG

$| \quad |$ $u = x^3-1 \Longrightarrow du = 3x^2\,dx$

for $x > 1$ the followin makes sense



$$= \frac{1}{3}\int \frac{3x^2}{\sqrt{x^3-1}}\,dx$$

$$= \frac{1}{3}\int (x^3-1)^{-\frac{1}{2}} \cdot 3x^2\,dx$$

$$= \frac{1}{3}\int u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{3} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}(x^3-1)^{\frac{1}{2}} + C$$

$$= \frac{2}{3}\sqrt{x^3-1} + C$$

5.5 #33: $u = \dfrac{1}{t} - 1$ $\quad \dfrac{1}{t} = t^{-1} - 1$

$du = -\dfrac{1}{t^2}\,dt$ $\quad du = -t^{-2}$

$= -\dfrac{1}{t^2}$