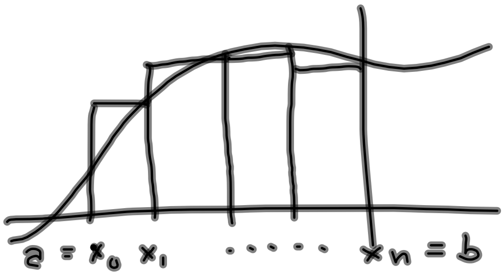


Right endpoints

$x_k = a + k\Delta x$

$= a + \frac{b-a}{n} k$

Area $\approx$ sum of the areas of the rectangles

$= f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$

$= \sum_{k=1}^{n} f(x_k)\Delta x$

(More rectangles $\Longrightarrow$ Better accuracy)

(h small $\Longrightarrow$ $\frac{f(x+h)-f(x)}{h}$ close to $f'(x)$ = ACTUAL)

If f is SMOOTH, then $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$

Take width $= \Delta x \longrightarrow 0$, then

$\sum f(x_k)\Delta x \longrightarrow$ Area

$\Delta x \longrightarrow 0 \iff n \longrightarrow \infty$ if Δx's are all the same size.

$\max\{\Delta x\}$ = "norm of the partitition"

= "mesh" " "

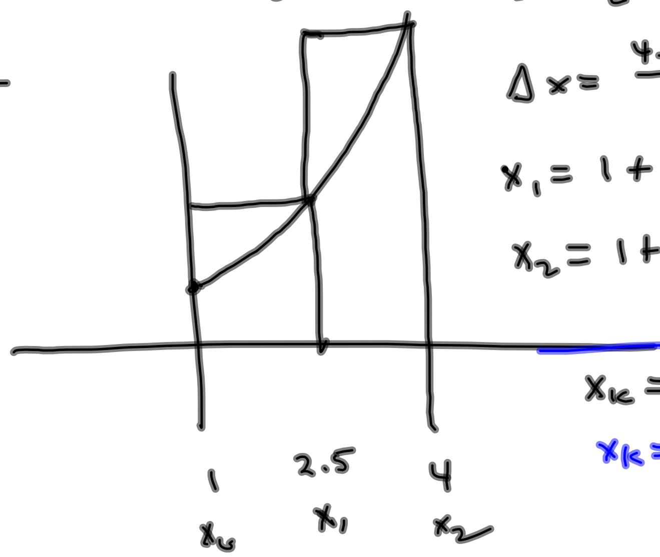
= size of largest grain that can slip thru the net.

$\frac{b-a}{n} = \Delta x$ for ALL our rectangles

as $n \to \infty$, $\Delta x = \frac{b-a}{n} \longrightarrow 0$

$f(x) = x^2$ on $[a,b] = [1,4]$

$n = 2$



$\Delta x = \frac{4-1}{2} = \frac{3}{2}$

$x_1 = 1 + \frac{3}{2} = \frac{5}{2} = 2.5$ ✓

$x_2 = 1 + 2 \cdot \frac{3}{2} = 4$ ✓

$x_k = 1 + k\Delta x$

$x_k = 1 + k \cdot \frac{3}{2}$

$= 1 + \frac{3k}{2}$

$$\sum_{k=1}^{2} f(x_k)\Delta x = \sum_{k=1}^{2} \left(1 + \frac{3k}{2}\right)^2 \cdot \frac{3}{2}$$

$$= \left(1 + \frac{3}{2}\right)^2\left(\frac{3}{2}\right) + \left(1 + \frac{3 \cdot 2}{2}\right)^2\left(\frac{3}{2}\right)$$

$$= \left(\frac{5}{2}\right)^2\left(\frac{3}{2}\right) + (4)^2\left(\frac{3}{2}\right)$$

$$= \frac{25}{4} \cdot \frac{3}{2} + 16 \cdot \frac{3}{2}$$

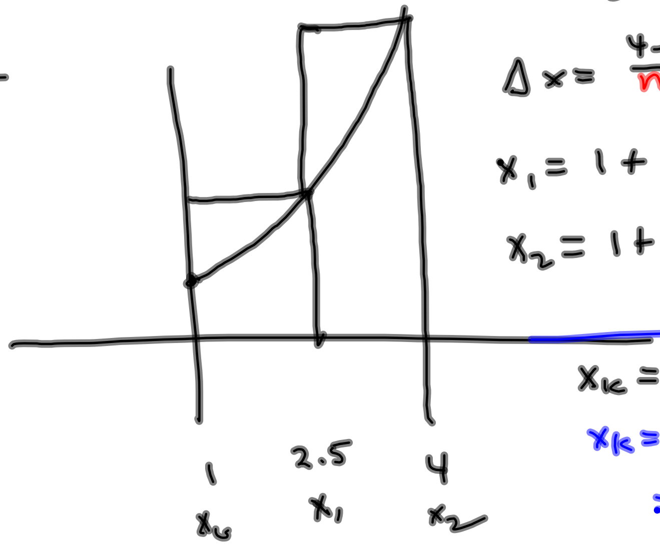
$$= \frac{75}{8} + 24 \cdot \frac{8}{8}$$

$$= \frac{75 + 192}{8} = \frac{267}{8}$$

$$\begin{array}{r} {}^{3}24 \\ \times\ 8 \\ \hline 192 \end{array}$$

$f(x) = x^2$ on $[a,b] = [1,4]$

$n=2$



$\Delta x = \frac{4-1}{n} = \frac{3}{n}$

$x_1 = 1 + \frac{3}{n} = 1 + \frac{3}{n}$

$x_2 = 1 + 2 \cdot \frac{3}{n} = 1 + \frac{6}{n}$

$x_k = 1 + k\Delta x$

$x_k = 1 + k \cdot \frac{3}{n}$

$= 1 + \frac{3k}{n}$

$$\sum_{k=1}^{2} f(x_k)\Delta x = \sum_{k=1}^{n} \left(1 + \frac{3k}{n}\right)^2 \cdot \frac{3}{n}$$

The idea is $n \to \infty$ gives us the EXACT area

Scratch: $\left(1 + \frac{3k}{n}\right)^2 = 1^2 + 2(1)\left(\frac{3k}{n}\right) + \left(\frac{3k}{n}\right)^2$

$(a+b)^2 = a^2 + 2ab + b^2$ $\quad = 1 + \frac{6k}{n} + \frac{9k^2}{n^2}$

This gives

$$\sum_{k=1}^{n} \left[\left(1 + \frac{6k}{n} + \frac{9k^2}{n^2}\right)\left(\frac{3}{n}\right)\right]$$

$$= \sum_{k=1}^{n} \left(\frac{3}{n} + \frac{18k}{n^2} + \frac{27k^2}{n^3} \right)$$

$$= \sum_{k=1}^{n} \frac{3}{n} + \sum_{k=1}^{n} \frac{18k}{n^2} + \sum_{k=1}^{n} \frac{27k^2}{n^3}$$

$$= \frac{3}{n} \sum_{k=1}^{n} 1 + \frac{18}{n^2} \sum_{k=1}^{n} k + \frac{27}{n^3} \sum_{k=1}^{n} k^2$$

$$= \frac{3}{n} \cdot n + \frac{18}{n^2} \cdot \frac{n^2 + \cdots}{2} + \frac{27}{n^3} \cdot \frac{n^3 + \cdots}{3}$$

$$\xrightarrow{n \to \infty} 3 + 9 + 9 = 21$$

$$\frac{18n^2 + 18n}{2n^2} \qquad \frac{27\,n(n+1)(2n+1)}{3n^3}$$

If $f(x)$ is cnt's, then the limit ALWAYS EXISTS.

$$\sum_{k=1}^{n} \frac{18k}{n^2} = \frac{18}{n^2}\cdot 1 + \frac{18}{n^2}\cdot 2 + \frac{18}{n^2}\cdot 3 + \cdots + \frac{18}{n^2}\cdot n$$

$$= \frac{18}{n^2}\left[1 + 2 + 3 + \cdots + n\right]$$

$$= \frac{18}{n^2}\sum_{k=1}^{n} k$$

$$\sum_{k=1}^{n} 1 = n$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2+n}{2} = \frac{n^2}{2} + \sim$$

Smaller stuff will vanish in the limit.

$$\int x\,dx = \frac{x^2}{2} + C$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + \sim}{6} = \frac{n^3}{3} + \sim$$

$$\int x^2\,dx = \frac{x^3}{3} + C$$

$$\frac{18}{n^2}\sum_{k=1}^{n} k = \frac{18}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= \frac{18n^2 + 18n}{2n^2}$$

$$\frac{18x^2+18x}{2x^2} \xrightarrow{n \to \infty} \frac{18}{2}$$

$$\frac{\cancel{x^2}\left(18 + \frac{18}{x}\right)}{2\cancel{x^2}} \xrightarrow{x \to \infty} \frac{18+0}{2}$$

FTC I

If f is cont^{s} on $[a,b]$ and $a < x < b$, then

$$g(x) = \int_a^x f(t)\,dt \text{ is Differentiable}$$

$$g'(x) = f(x)$$

$$h(x) = g(3x^2) = \int_a^{3x^2} f(t)\,dt$$

$u = 3x^2$

$\frac{du}{dx} = 6x$

$$\frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = f(3x^2) \cdot 6x$$

$$g(x) = \boxed{\int_0^x \sin(t)\,dt}$$

$$\sum_{k=1}^{n} * = \sum_{j=1}^{n} *$$

$$g'(x) = \sin(x)$$

$$g(3x^2) = \int_0^{3x^2} \sin(r)\,dr$$

$u = 3x^2$

$\frac{du}{dx} = 6x$ $\quad \frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = \sin(3x^2) \cdot 6x$

$$\frac{d}{dx}\left[x^2-5\right]=2x$$

$$\frac{d}{dx}\left[(x^2-5)^{30}\right]$$

$$=\frac{d}{d(x^2-5)}\left[(x^2-5)^{30}\right]\cdot\frac{d(x^2-5)}{dx}$$

$$\frac{d}{dx}\left[x^{30}\right]=30x^{29}$$

$$=\frac{d}{du}\left[u^{30}\right]=30u^{29}$$

$$\frac{d}{dx}\left[(x^2-5)^{30}\right]=\frac{d}{dx}\left[g(u(x))\right], \text{where}$$

$$g(x)=x^{30}$$

$$u(x)=x^2-5$$

$$u=x^2-5$$

$$\frac{du}{dx}=2x$$

$$\frac{dg}{du}=30u^{29}$$

$$\frac{du}{dx}=2x$$

$$\frac{dg}{du}\cdot\frac{du}{dx}=(30u^{29})(2x)$$

$$=30(x^2-5)^{29}(2x)$$

Oct 25-1:06 PM

$$\frac{d}{dx}\int_0^x \cos^2\theta \sin^2\theta \, d\theta$$

$$= \cos^2 x \sin^2 x$$

$$\frac{d}{dx}\int_0^{\tan x} \cos^2\theta \sin^2\theta$$

$$= \underbrace{\cos^2(\tan x)\sin^2(\tan x)}_{\frac{dg}{du}} \cdot \underbrace{\sec^2 x}_{\frac{du}{dx}}$$

where $g = \cos^2 u \sin^2 u$

and $u = \tan x$

$$\boxed{\frac{dg}{d(\tan x)}} \cdot \frac{d(\tan x)}{dx}$$

$$\frac{dg}{du}$$