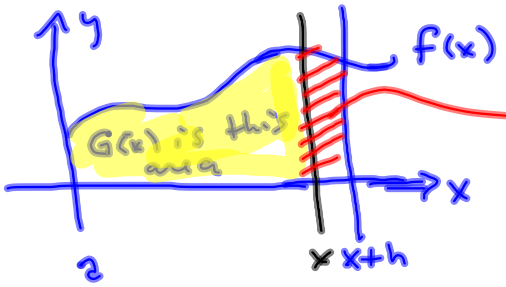
§5.4

FTC I

$G(x) = \int_a^x f(t)\,dt$ is a function of x.



$$\int_a^{x+h} - \int_a^x = \int_x^{x+h}$$

If f is continuous on $[a,b]$ and $x \in [a,b]$, with

$$G(x) = \int_a^x f(t)\,dt \implies$$

$$G'(x) = \frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Proved last time, using MVT for integrals

$\exists\, c \in [a,b]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(w)\,dw$$

$\exists\, c \in [x, x+h] \ni$

$$f(c) = \boxed{\frac{1}{h}\int_x^{x+h} f(t)\,dt} \xrightarrow{h \to 0} f(x)$$

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right]$$

$$\frac{\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt}{h}$$

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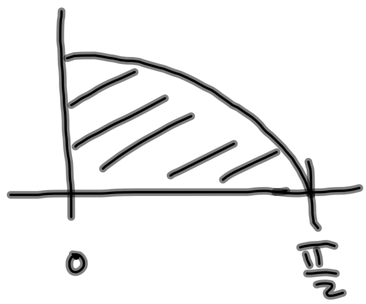

$y = \cos x$

$$\int_0^{\frac{\pi}{2}} \cos(x)\,dx$$

$$= \Big[\sin(x)\Big]_0^{\frac{\pi}{2}} =$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin(0)$$

$$= 1 - 0 = 1$$

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$$G(x) = \int_0^x (t^2+1)\,dt \implies$$

what's

$$\frac{d}{dx}\int_a^{x^4} (t^2+1)\,dt$$

$$= \left((x^4)^2+1\right)\cdot 4x^3$$

$$= \frac{dG}{du}\cdot\frac{du}{dx}, \text{ where } u=x^4$$

$$G'(x) = (x^2+1)\cdot\frac{dx}{dx}$$

Recall Chain Rule

$$\frac{d}{dx}\left[G(x^4)\right] = \frac{dG}{d(x^4)}\cdot\frac{d}{dx}(x^4)$$

$$u = x^4 \qquad \frac{dG}{du}\cdot\frac{du}{dx} = G'(u)\,u'(x)$$

$$\frac{d}{dx}\left[\sin(x^4)\right] = \cos(x^4)\cdot 4x^3$$

$$= \frac{d\sin(x^4)}{d(x^4)}\cdot\frac{d(x^4)}{dx}$$

$$= \frac{dG}{du}\cdot\frac{du}{dx}$$

$$= G'(x^4)\cdot\frac{d}{dx}\left[x^4\right]$$

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$$\frac{d}{dx}\int_0^{\sin x} \frac{w^2-5w}{\sqrt{w+1}}\,dw$$

The w is a dummy variable

$$= \frac{\sin^2 x - 5\sin x}{\sqrt{\sin x + 1}} \cdot \cos x$$

$$\int_0^3 (t^2-5)\,dt = \int_0^3 (x^2-5)\,dx$$

$$\sum_{k=1}^{5}(3k-2) = \sum_{j=1}^{5}(3j-2)$$

Index variable is a dummy variable.

Only has meaning INSIDE the Σ or the $\int$.

MUST STAY INSIDE the Σ or the $\int$.

$$\boxed{\frac{d}{dx}\left[\sin(x^2-2)\right] = \cos(x^2-2)\cdot 2x}$$

$$\frac{d}{dx}\left[\sin^2(x^2-2)\right] = 2\sin(x^2-2)\cdot\cos(x^2-2)\cdot 2x$$

$$\int \underbrace{\cos(x^2-2)\cdot 2x}\,dx = \sin(x^2-2) + C$$

we recognize this as the result of applying the chain rule to $\sin(x^2-2)$

$$\int x\cos(x^2-2)\,dx = \frac{1}{2}\int \cos(x^2-2)\cdot 2x\,dx$$

Let $u = x^2-2$. Then

$\frac{du}{dx} = 2x$ and so

$du = 2x\,dx$

$$= \frac{1}{2}\int \cos(u)\,du$$

$$= \frac{1}{2}\sin(u) + C$$

$$= \frac{1}{2}\sin(x^2-2) + C$$

$$\int (x^4-x^2)^{2/3}\,dx$$ have $u^{2/3}$ but no du. can't do.

$$\int (x^4-x^2)^{2/3}(4x^3-2x)\,dx = \int u^{\frac{2}{3}}\,du$$

(Let $u = x^4 - x^2$
$\Longrightarrow du = (4x^3-2x)\,dx$)

$$= \frac{u^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3}{5}u^{\frac{5}{3}} + C = \frac{3}{5}(x^4-x^2)^{\frac{5}{3}} + C$$

u-substitution − Let u = what's inside and build du.

only manipulation allowed is multiply & divide by a constant.

x

$$\int \sin(x^2+1)\cdot\frac{2x}{2x}\,dx$$

$$= \frac{1}{2x}\int \sin(x^2+1)\cdot 2x\,dx$$

$$\int \sin(x^2+1)\,dx = \frac{1}{2x}\int \sin(x^2+1)\cdot 2x\,dx$$

$u = x^2+1$
$du = 2x\,dx$

NO!

x is dummy variable. Can't pass the dummy thru the $\int$-sign.

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$$\int \sin(x^3)\, x^2\, dx \qquad \begin{array}{l} u = x^3 \\ du = 3x^2 dx \end{array}$$

$$\frac{1}{3}\int \sin(x^3)\, 3x^2\, dx$$

$$= \frac{1}{3}\left(-\cos(x^3)\right) + C$$

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