Excel File for Newton's Method:
I'll ZIP it and try THAT.

$$1+2+3+\cdots+n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

5050

$$\sum_{k=1}^{100} k = \frac{100(101)}{2} = \frac{10100}{2} = 5050$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

why?!

Because

$$\underbrace{\text{Area} = \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x_k}$$

Means there is an N so that, given $\varepsilon > 0$

$$\left| \int_a^b f(x)\,dx - \sum_{k=1}^{n} f(x_k)\Delta x_k \right| < \varepsilon$$

whenever $n \geq N$.

The difference between the integral and the finite sum can be made arbitrarily small by taking n sufficiently large.

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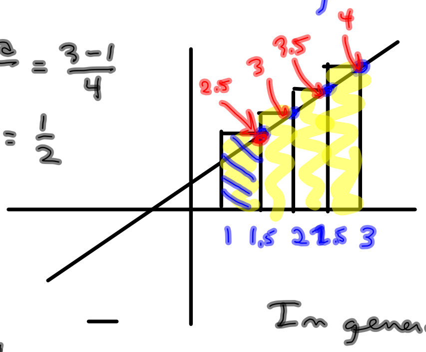
$f(x) = x + 1$

Find the area under $f(x)$ on the interval $[a,b] = [1,3]$ by taking the limit of a Riemann Sum.

For practice, use right endpoints and $n = 4$ rectangles $(4+3+3.5+2.5)\frac{1}{2} = \frac{13}{2}$ ✓

$\Delta x = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$



$x_0 = a = 1$

$x_1 = a + \Delta x = 1 + \frac{1}{2}$

$x_2 = x_1 + \frac{1}{2} = a + \frac{1}{2} + \frac{1}{2} = a + 2\Delta x = 1 + 2\cdot\frac{1}{2} = 1 + 1 = 2$

$x_3 = 1 + 3\Delta x = 1 + \frac{3}{2}$

$x_4 = 1 + 4\Delta x$

In general

$x_k = a + k\Delta x = a + k\cdot\frac{b-a}{n} = 1 + k\cdot\frac{1}{2}$

$\text{Area} \approx \sum_{k=1}^{4} f(x_k)\Delta x_k = \sum_{k=1}^{4} (x_k + 1)\cdot\frac{1}{2}$

$= \sum_{k=1}^{4}\left(1 + \frac{k}{2} + 1\right)\cdot\frac{1}{2} = \frac{1}{2}\sum_{k=1}^{4}\frac{k+4}{2}$

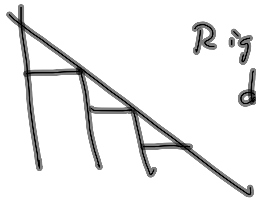
$= \frac{1}{2}\sum_{k=1}^{4}\frac{1}{2}(k+4)$

$= \frac{1}{4}\sum_{k=1}^{4}(k+4) = \frac{1}{4}\left[(1+4)+(2+4)+(3+4)+(4+4)\right]$

$= \frac{1}{4}[5+6+7+8] = \frac{1}{4}[26] = \frac{13}{2}$

$A \approx \frac{13}{2}$ is an overestimate

(Right endpoints + Increasing Function)



Right endpoints & decreasing $\Longrightarrow$ underestimate.

n rectangles:

$$\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$$

$$x_k = a + k\Delta x = 1 + k \cdot \frac{2}{n} = \frac{n+2k}{n} = 1 + \frac{2k}{n}$$

$$f(x_k) = x_k + 1 = 1 + \frac{2k}{n} + 1 = 2 + \frac{2k}{n}$$

$$\therefore \sum_{k=1}^{n} f(x_k)\Delta x_k = \sum_{k=1}^{n} \left(2 + \frac{2k}{n}\right)\cdot \frac{2}{n}$$

$$= \frac{2}{n} \sum_{k=1}^{n} \left(2 + \frac{2k}{n}\right)$$

$$= \frac{2}{n}\left[\sum_{k=1}^{n} 2 + \sum_{k=1}^{n} \frac{2k}{n}\right]$$

$$= \frac{2}{n}\left[\sum_{k=1}^{n} 2 + \frac{2}{n}\sum_{k=1}^{n} k\right]$$

$$= \frac{2}{n}\left[2n + \frac{2}{n}\cdot\frac{n(n+1)}{2}\right] =$$

$$y = \frac{2x^2+2x}{x^2}$$

$$\xrightarrow{x\to\infty} \frac{2x^2}{x^2} = 2$$

$$= \frac{2}{n}\cdot 2n + \frac{2}{n}\cdot\frac{\cancel{2}}{n}\cdot\frac{n(n+1)}{\cancel{2}}$$

$$= 4 + \frac{2n(n+1)}{n^2} = 4 + \frac{2n^2+2n}{n^2}$$

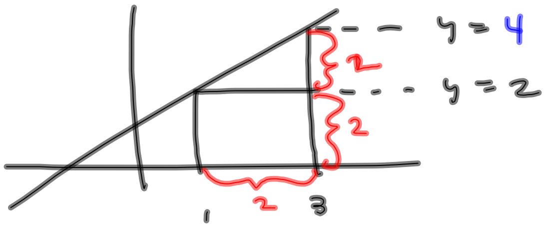
$$\xrightarrow{n\to\infty} 4 + 2 = 6 = \int_1^3 (x+1)\,dx$$

$$\sum_{k=1}^{n} 2 = \underbrace{2+2+2+\cdots+2}_{n \text{ of 'em}}$$

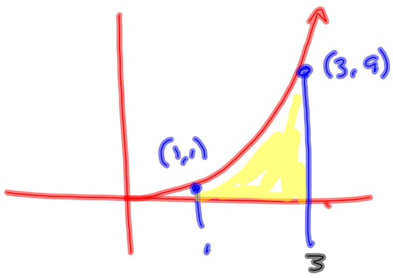
$$1+2+3+4+5+6 =$$

$$1+3+5+2+4+6$$

As a geometry prob:



$$2 \cdot 2 + \tfrac{1}{2} \cdot 2 \cdot 2 = 4 + 2 = 6$$



$y = x^2$ from $x = 1$ to $x = 3$

$$x_k = 1 + k \cdot \frac{3-1}{n} = 1 + \frac{2k}{n}$$

$$\Delta x = \frac{2}{n}$$

$$\sum_{k=1}^{n} f(x_k)\,\Delta x = \sum_{k=1}^{n} \left(1 + \frac{2k}{n}\right)^2 \cdot \frac{2}{n}$$

$$= \frac{2}{n} \sum_{k=1}^{n} \left(1 + \frac{4k}{n} + \frac{4k^2}{n^2}\right)$$

$$= \frac{2}{n}\left[\sum_{k=1}^{n} 1 + \frac{4}{n}\sum_{k=1}^{n} k + \frac{4}{n^2}\sum_{k=1}^{n} k^2\right]$$

$$= \frac{2}{n}\left[n + \frac{4}{n} \cdot \frac{n(n+1)}{2} + \frac{4}{n^2} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$= 2 + \cancel{\frac{2}{n}} \cdot \frac{4}{n} \cdot \left(\frac{n^2+n}{\cancel{2}}\right) + \frac{2}{n} \cdot \frac{4}{n^2} \cdot \frac{(n^2+n)(2n+1)}{6}$$

$$= 2 + \frac{4n^2+4n}{n^2} + \frac{8}{6n^3}(2n^3 + \text{smaller})$$

$$\xrightarrow{n \to \infty} 2 + 4 + \frac{16}{6} = 6 + \frac{8}{3} = \boxed{\frac{26}{3}}$$

$$\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{3^3}{3} - \frac{1^3}{3} = \frac{27-1}{3} =$$

Shortcut $(n^2+n)(2n+1)$

$= 2n^3 + \text{smaller}$

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Next time:

Take notes on boxes

§5.1 - 5.3

Newton's : Excel will be e-mailed to you.

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