For Newton Demo, go to

http://www.harryzaims.com/201-all/201-fall-12/notes/chapter-04/121018-newton-demo/

and click on

121018-newton-demo.html

OR just go here:

http://www.harryzaims.com/201-all/201-fall-12/notes/chapter-04/121018-newton-demo/121018-newton-demo.html

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Newton's Method Assignment:

Due Tuesday.

4.5 no sooner than Monday

Next time: Starting C5

Area under a curve.

Aiming at Fundamental Theorem of Calculus I, II

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Why $\int f(x)\,dx$?

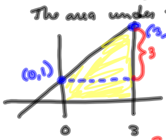
Area under velocity curve is DISTANCE

$v(x) = x+1$

$\int (x+1)\,dx = \frac{1}{2}x^2 + x + C = F(x)$

How far did the guy travel, from $x=0$ to $x=3$?

The area under the velocity curve is:



Δ on $\square$ =

$\frac{1}{2}\cdot 3\cdot 3 + 3(1) =$

$= \frac{9}{2} + 3 = \frac{9+6}{2} = \frac{15}{2}$

Area under curve is $\int_0^3 (x+1)\,dx$ Fundamental Theorem of Calculus.

$= \left[\frac{1}{2}x^2 + x + C\right]_0^3 = F(3) - F(0)$

$= \frac{1}{2}(3)^2 + 3 + c - \left(\frac{1}{2}(0)^2 + 0 + c\right)$

$= \frac{9}{2} + 3 + \underline{c - c} = \frac{15}{2}$

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Example Area under $f(x) = x^3 + 2x^2 - 3$ from $x=0$ to $x=2$

$\int_0^2 (x^3+2x^2-3)\,dx$

$= \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - 3x\right]_0^2$

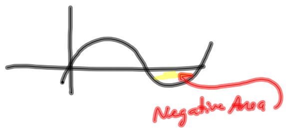
$= \frac{2^4}{4} + \frac{2(2^3)}{3} - 3(2) - [0 + 0 - 0]$

$= 4 + \frac{16}{3} - 6$

$= \frac{12+16-18}{3}$

$= \frac{10}{3}$ This is SIGNED area.

When $f(x) < 0$, Area is NEGATIVE.



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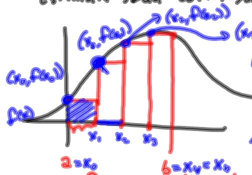
The Lead-in:

Estimate Area with rectangles.

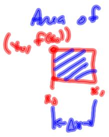


LEFT END POINTS

$n=4$ rectangles.

Find area from a to b under the curve.

Area of the rectangles:



Easy Δx is $\frac{b-a}{n} = \frac{b-a}{4}$

$= f(x_0)\Delta x$

$= f(x_0)\frac{b-a}{n} = f(x_0)\frac{b-a}{4}$

Total: Left endpts:

$f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$

$= \sum_{k=0}^{3} f(x_k)\Delta x$

Right Endpts

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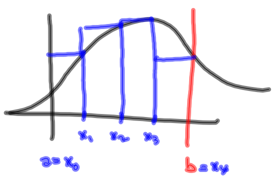

Righties!

Area $\approx f(x_1)\Delta x + \dots + f(x_4)\Delta x$

$= \sum_{k=1}^{4} f(x_k)\Delta x$

Newton Says: More Rectangles, better Estimate

And if $f(x)$ is continuous, then

$$\underbrace{\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x}_{\int_a^b f(x)\,dx} = \text{Area exactly}$$

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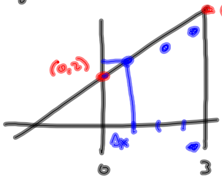
Find the Exact area under

$y = x+2$ from $x=0$ to $x=3$



Use Right Endpoints

$n = \#$ of rectangles

$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$

$x_1 = 0 + \Delta x = \frac{3}{n}$

$x_2 = x_1 + \Delta x = \frac{3}{n} + \frac{3}{n} = \frac{6}{n} = \frac{3}{n}\cdot 2$

$x_3 = x_2 + \Delta x = x_2 + \frac{3}{n} = 3\cdot\frac{3}{n}$

$x_1 = 0 + \frac{3}{n}$ $x_2 = \frac{3}{n} + \frac{3}{n} = 2\frac{3}{n}$ $x_3 = 2\cdot\frac{3}{n} + \frac{3}{n}$

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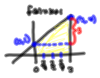
$f(x) = x+1$



Fine. We'll do this one, with $n = 4$ rectangles.

$2 + \frac{3}{4}$

$\frac{3}{2} + \frac{3}{4} = \frac{9}{4}$

$x_1 = 0 + 1\Delta x = 0 + 1\cdot\frac{3}{4}$

$x_2 = \frac{3}{4} + 1\Delta x = \frac{3}{4} + 1\cdot\frac{3}{4}$

$= \frac{6}{4} = \frac{3}{2}$

$\Delta x = \frac{3-0}{4} = \frac{3}{4}$

$x_3 = \frac{3}{2} + \frac{3}{4} = \frac{9}{4}$

Area $\approx f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_4)\Delta x$

$x_k = 0 + k\Delta x$

$= k\cdot\frac{3}{4} = \frac{3}{4}k$

$f(x_k) = \underline{x_k + 1} = \frac{3}{4}k + 1$

$\sum_{k=1}^{4}\left(\frac{3}{4}k+1\right)\frac{3}{4}$ $f(x) = x+1$

$\left(\frac{3}{4}+1\right)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\cdot 2+1\right)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\cdot 3+1\right)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\cdot 4+1\right)\left(\frac{3}{4}\right)$

Now, for arbitrary n

$x_k = x_0 + k\cdot\Delta x$

$= 0 + k\cdot\frac{3}{n} = \frac{3k}{n}$

$f(x_k) = x_k + 1 = \frac{3k}{n} + 1$

$f(x_k)\Delta x = \left(\frac{3k}{n}+1\right)\left(\frac{3}{n}\right)$

$= \left(\frac{3k+n}{n}\right)\left(\frac{3}{n}\right) = \frac{9k+3n}{n^2}$

$\Rightarrow$ Area $\approx \sum_{k=1}^{n}\frac{9k+3n}{n^2}$

$\xrightarrow{n\to\infty} \int_0^3 (x+1)\,dx$

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$1 + 2 + 3 + \cdots + n$

$1 + 2 + \cdots + 100$

$\frac{n}{2}$

$\frac{100}{2}(101)$

$= \frac{n(n+1)}{2}$

$\sum k^2$

$\sum k^3$

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