Quizzes/Other: 40%

Quiz 20%

Practice Test 20%

midterm 30%

Final 30%

§3.6 Rational Functions $\frac{P(x)}{Q(x)}$

zeros $f(x)=0$ (Numerator $=0$) Find x

y-int $f(0)$

Domain $Q(x)\neq 0$

vertical Asymptotes $\boxed{Q(x)=0 \ \& \ P(x)\neq 0}$

Horizontal Asymptotes $|x| \to$ BIG

Proper & Improper Functions

Oblique Asymptotes.

Improper

$Q(x)=0$ AND $P(x)=0$

HOLE

Oct 19-10:15 AM

Graph $\dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$

It's <u>Proper</u>

$\frac{1}{3}$ is proper

$\frac{4}{3}$ is improper

$\dfrac{x}{x^2+2}$, $\dfrac{1}{x^2-1}$, $\dfrac{3x^3+7x}{5x^4-7x^3+29x}$ are proper

$\dfrac{x^2+2}{x}$ is improper (Degree of denominator is greater than the degree of the numerator.

When <u>PROPER</u>,

$\boxed{\text{horizontal asymptote } y=0}$

(Denominator Dominates)

From last time:

$f(x) = \dfrac{1}{(x-1)(x+1)}$

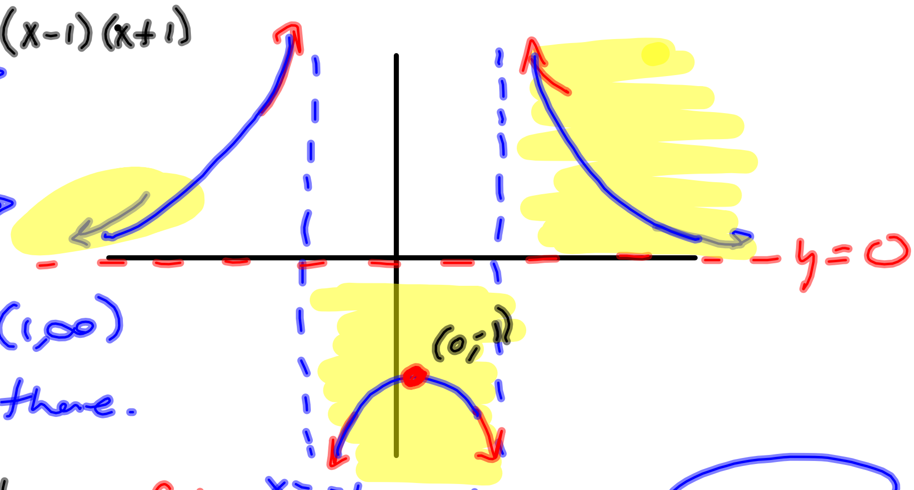
x-intercepts:

<u>NONE</u> $1 \neq 0$

$\mathcal{D} = \{x \mid x \neq \pm 1\}$

$= (-\infty,-1) \cup (-1,1) \cup (1,\infty)$

Numerator $\neq 0$ there.



	Test	f(x)		
$(-\infty,-1)$	$x=-2$	$\dfrac{1}{(-2-1)(-2+1)} = \dfrac{1}{(-3)(-1)} = \dfrac{1}{3}$	+
$(-1,1)$	$x=0$	$\dfrac{1}{(0-1)(0+1)} = \dfrac{1}{(-1)(1)} = -1$	−
$(1,\infty)$	$x=2$	$\dfrac{1}{(2-1)(2+1)} = \dfrac{1}{(1)(3)} = \dfrac{1}{3}$	+

Oct 19-10:21 AM

Horizontal Asymptotes

$\frac{x}{x^3+2}$, $\frac{1}{x^2+27x}$, ... Proper $\Rightarrow y=0$

$$\frac{2x^3+5x^2-1}{7x^3-4x+25} \qquad y=\frac{2}{7}$$

$$\frac{2x^3+5x^7-2x+1}{6x^7-2200x^3-10^6x+11} \qquad y=\frac{5}{6}$$

when $|x|$ is BIG, the high powers Dominate.

$$\xrightarrow{|x|\to\infty} \frac{2x^3}{7x^3}=\frac{2}{7}$$

$$\frac{5x^7+\text{smaller}}{6x^7+\text{smaller}} \xrightarrow{|x|\to} \frac{5x^7}{6x^7}=\frac{5}{6}$$

Oct 19-10:36 AM

$$R(x) = \frac{2x^2 \cancel{-7x-15}}{3x^2 \cancel{+5x+2}} = \frac{(2x+3)(x-5)}{(3x+2)(x+1)}$$

$(2x+3)(x-5)$
$= 2x^2-7x-15$
$(3x+2)(x+1)$
$= 3x^2+5x+2$

x-int: $(-\frac{3}{2}, 0), (5, 0)$

y-int: $(0, -\frac{15}{2})$

$\mathcal{D} = \{x \mid x \neq -\frac{2}{3} \text{ AND } x \neq -1\}$

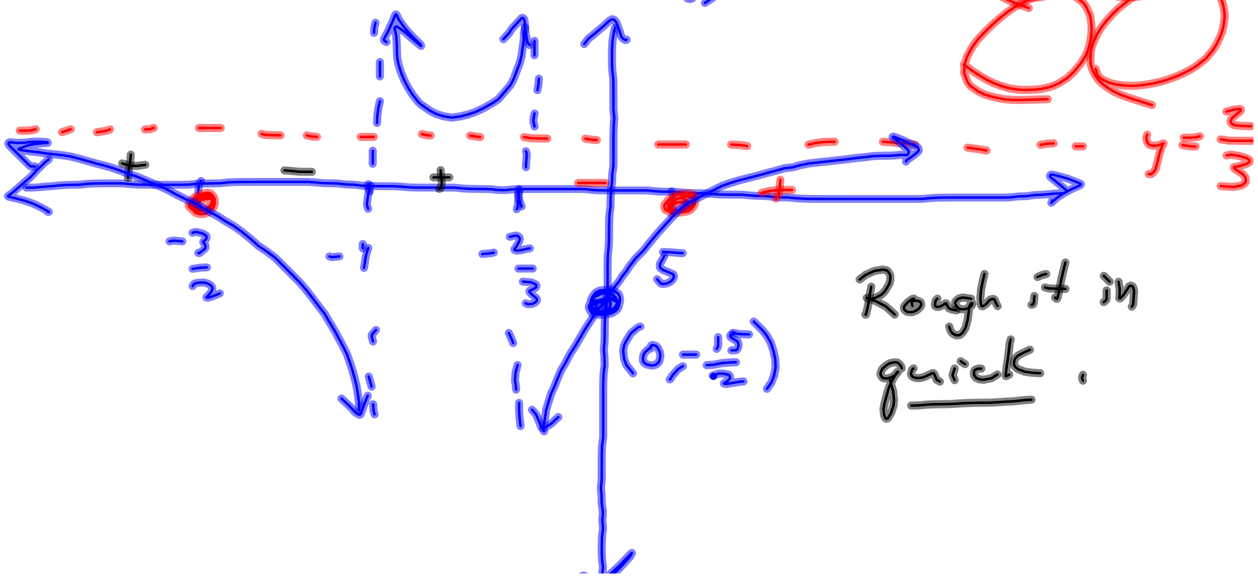
Are these zeros of the numerator?
No $\Longrightarrow$ Vertical Asymptotes

H.A.: $y = \frac{2}{3}$ $\qquad \frac{2x^2 + \text{small}}{3x^2 + \text{small}} \xrightarrow{|x| \to \infty} \frac{2}{3}$

$$= \frac{(2x+3)^1(x-5)^1}{(3x+2)^1(x+1)^1}$$

$x = -3/2, 5, -\frac{2}{3}, -1$ are where it can change sign.

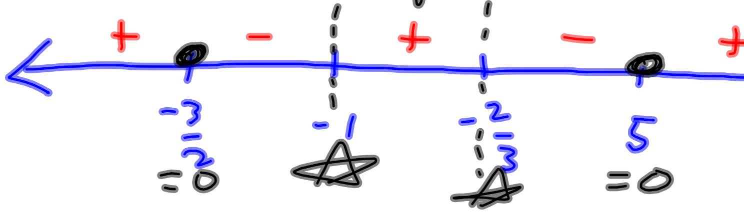
$-\frac{3}{2}, -1, -\frac{2}{3}, 5$



Rough it in quick.

Solve $\dfrac{(2x+3)(x-5)}{(3x+2)(x+1)} > 0$

Sign pattern:



3.6, 4.1, 4.2
Next time.

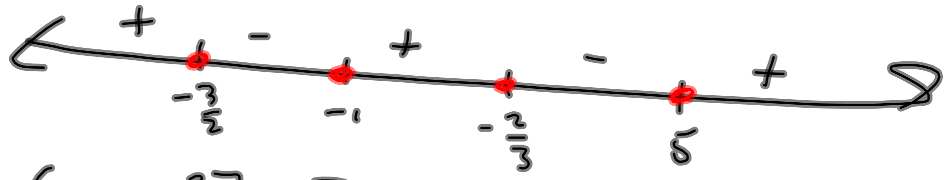
$(-\infty, -\frac{3}{2}) \cup (-1, -\frac{2}{3}) \cup (5, \infty)$

$\dfrac{(2x+3)(x-5)}{(3x+2)(x+1)} \geq 0$

$(-\infty, -\frac{3}{2}] \cup (-1, -\frac{2}{3}) \cup [5, \infty)$

Fine point: Domain plays a role.

$(2x+3)(x-5)(3x+2)(x+1) \geq 0$



$(-\infty, -\frac{3}{2}] \cup [-1, -\frac{2}{3}] \cup [5, \infty)$