Questions §4.3 Today: Some 4.5 & 4.6

(25) $3r^3+16r = r(3r^2+16)$

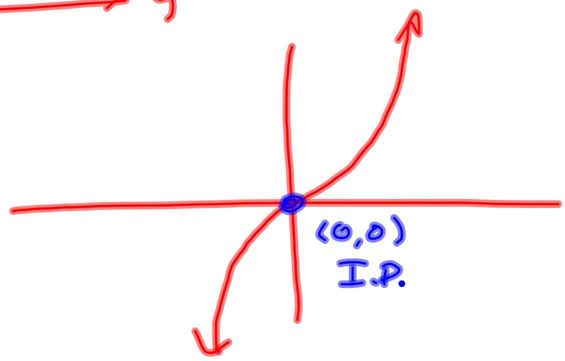
$r=0$ is only x-int.

$(0,0)$

$y' = 9r^2+16$ >0 always

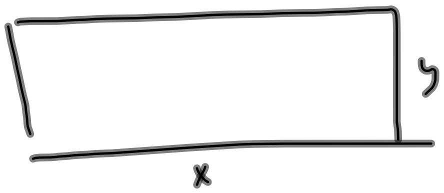
$y'' = 18r$ I.P. @ $r=0 \rightsquigarrow (0,0)$ is I.P.

$\overset{-}{\longleftarrow} \mid_{0} \overset{+}{\longrightarrow}$ y''



§4.5 #26 For funzies

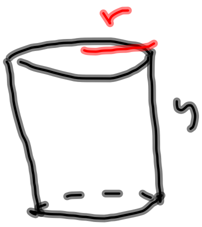
(a)



$2x+2y = 36$ cm = Perimeter

$x+y=18$

$y = 18-x$



x = circumference $= 2\pi r \rightarrow r = \frac{x}{2\pi}$

what x & y give max vol.?

V = volume $= \pi r^2 h$

$= \pi r^2 y$

$= \pi \left(\frac{x}{2\pi}\right)^2 y$

$= \pi \left(\frac{x}{2\pi}\right)^2 (18-x)$

$= \frac{x^2}{4\pi}(18-x)$

$= \frac{1}{4\pi} x^2 (18-x)$

Let $a = \frac{1}{4\pi}$

$V = ax^2(18-x) = 18ax^2 - ax^3$

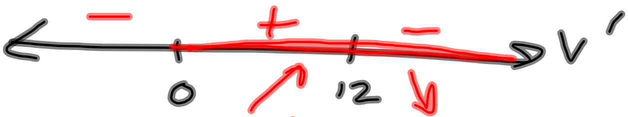
$\frac{dV}{dx} = 2ax(18-x) + ax^2(-1)$

$= 36ax - 3ax^2$ SET 0

4π $\quad 3ax(12-x) = 0$

$x = 0, 12$

$x = 12$'s my guess $\quad 1/4\pi \quad \frac{1}{4}\pi \quad \frac{1}{4\pi}$



1st deriv test says max @ $x=12$

$$\frac{d^2V}{dx^2} = V'' = 36a - 6ax = \boxed{6a(6-x)}$$

$+ \quad | \quad -$ $\quad V''$

$\cup \quad 6 \quad \cap$

$6\frac{1}{4\pi}(6-12)$

$V''(12) < 0$ $\qquad V''(12) < 0$

$\Rightarrow$ Max

2^{nd} derivative test. is good backup.

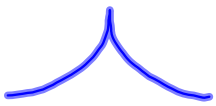
what if

$- \quad | \quad +$

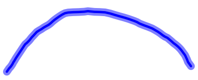
6

So we made a boo-boo!

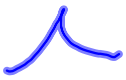
Then at $x=12$ we have



instead of

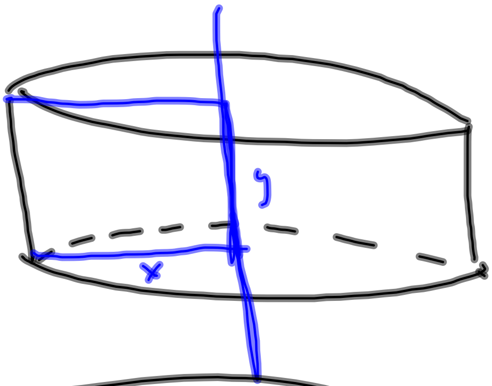


By $V'=0$ @ $x=12$, so



is impossible!

§4.5 26b



What values of x & y maximize the volume?

$2x+2y=36$

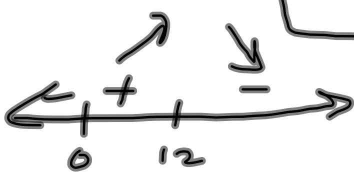
$$V=\pi r^2 h = \pi x^2 y = \pi(18-y)^2 y = \pi x^2(18-x)$$

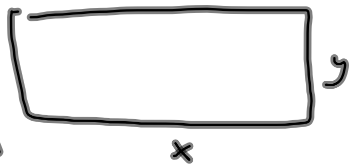
your choice

$$V = 18\pi x^2 - \pi x^3 = \pi\left[18x^2 - x^3\right]$$

$\cancel{= \pi x^2\left[18-x\right]}$

$$\frac{dV}{dx} = \pi\left[36x - 3x^2\right] = 3\pi\left[12x - x^2\right] = 3\pi x\left[12-x\right]$$



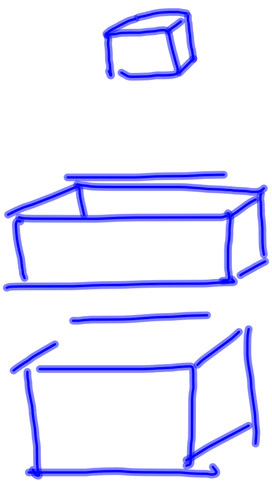


$x=12$!? why $x=12$, again?

Hmmmm Interesting.

Not sure.

Maximizes area of rectangle?



$$2x+2y=36$$

$$x+y=18$$

$$A=xy=x(18-x)=18x-x^2$$

$$\frac{dA}{dx}=18-2x$$

$$=2(9-x)$$

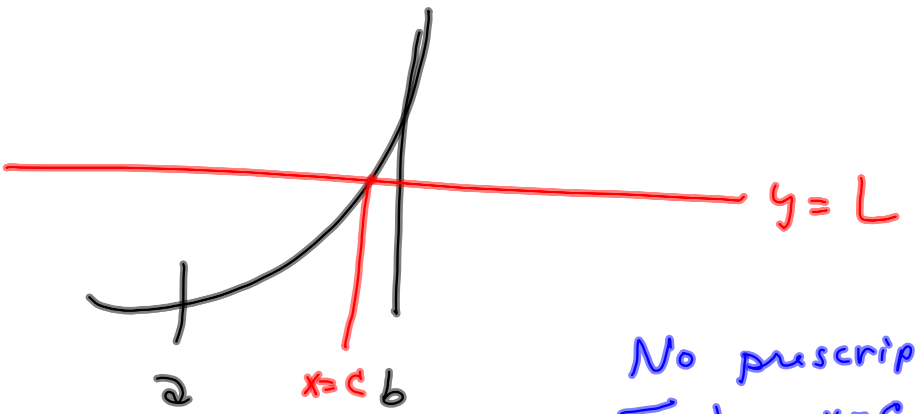
$x=9$ maximizes area, so

$y=18-9=9$, also.

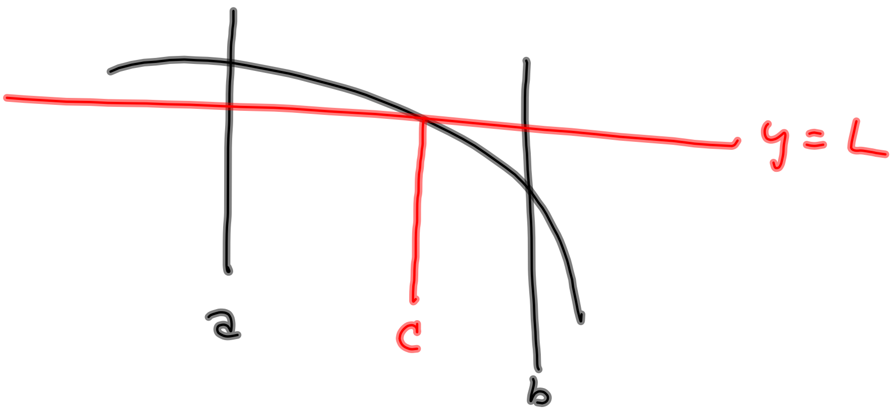
Rose says maximizing radius wasn't it, either.
~~Hmmmm~~ Not sure why $x=12$ did it
for both.

4.6 Newton's Method:

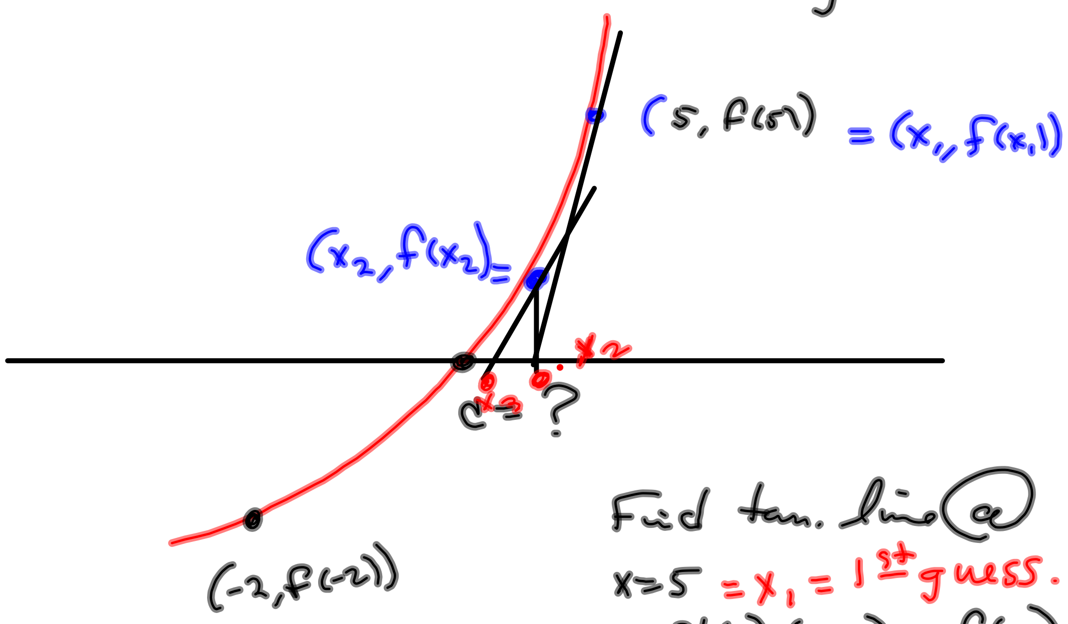
Recall IVT says
f cnts on $[a,b]$
$f(a) \neq f(b)$ & L is between those
two, then $\exists\, c \in (a,b) \ni f(c) = L$



No prescription for
<u>Finding</u> $x=c$!



Newton's Method for finding zeros.



Find tan. line @ $x=5 = x_1 = 1^{st}$ guess.

$$y = f'(5)(x-5) + f(5)$$

Find its x-int:

$$f'(5)(x-5) + f(5) = 0 \Longrightarrow$$

$$f'(5)(x-5) = -f(5)$$

$$x - 5 = \frac{-f(5)}{f'(5)}$$

$$x_2 = x = 5 - \frac{f(5)}{f'(5)} = 2^{nd} \text{ guess}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$\vdots$

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

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$(x-2)(x+5)$

$= x^2+3x-10$

$x=2$ & $x=-5$ are zeros.

Newton's with 1st guess @ $x=12$

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

$x_1 = 12$ is seed value

$$x_n = x_{n-1} - \frac{x_{n-1}^2+3x_{n-1}-10}{2x_{n-1}+3}$$

$$x_2 = 12 - \frac{12^2+3(12)-10}{2(12)+3} = \frac{144+36-10}{27}$$

$$= \frac{170}{27} \approx 6.296 = x_2$$

$$x_3 = \frac{x_2^2+3x_2-10}{2x_2+3}$$

	A	B	C	D	E
1	xn	f(xn)	f'(xn)	xn-f(xn)/f'(xn)	
2	12	170	27	5.703704	
3	5.703704	39.64335	14.40741	2.952109	
4	2.952109	7.571274	8.904218	2.101807	
5	2.101807	0.723013	7.203614	2.001439	
6	2.001439	0.010074	7.002878	2	
7	2	2.07E-06	7.000001	2	
8	2	8.7E-14	7	2	
9	2	0	7	2	
10	2	0	7	2	
11	2	0	7	2	
12	2	0	7	2	

Zeros found pretty in very few steps

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

OR

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$