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$$f(x) = x^4 + 2x^3$$

$$f'(x) = 4x^3 + 6x^2 \stackrel{SEF}{=} 0$$

$$2x^2(2x+3) = 0$$

$$x \in \{0, -\frac{3}{2}\}$$

$$f(0) = 0 \rightsquigarrow (0, 0) \text{ Poss Max}$$

$$f(-\frac{3}{2}) = -\frac{27}{16} \rightsquigarrow (-\frac{3}{2}, -\frac{27}{16}) \text{ Poss min}$$

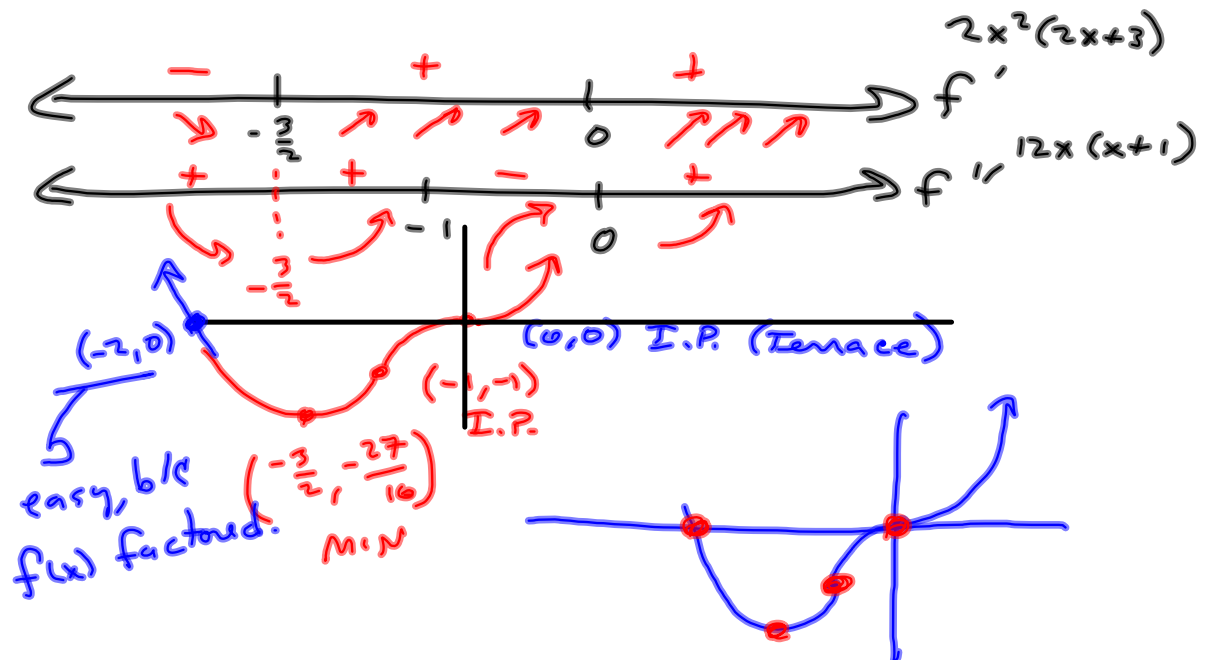
$$f''(x) = 12x^2 + 12x \stackrel{SEF}{=} 0$$

$$12x(x+1) = 0$$

$$x \in \{0, -1\}$$

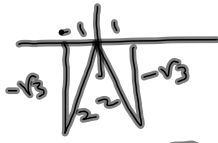
$$f(0) = 0 \rightsquigarrow (0, 0) \text{ IP Poss}$$

$$f(-1) = -1 \rightsquigarrow (-1, -1) \text{ IP Poss}$$

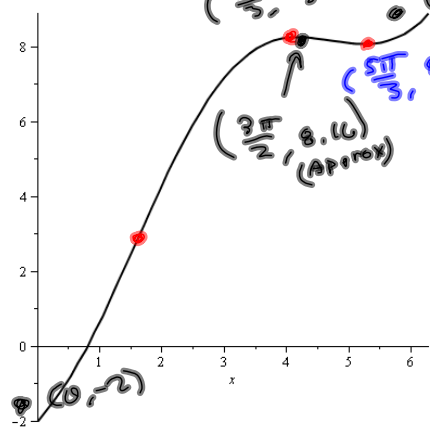


(25) $f(x) = \sqrt{3}x - 2\cos x$ $0 \leq x \leq 2\pi$
 $f'(x) = \sqrt{3} + 2\sin x \stackrel{SET}{=} 0$

$\sin x = -\frac{\sqrt{3}}{2}$



$\frac{2\pi - \pi}{2} = \frac{\pi}{2}$
 $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$



$f(\frac{4\pi}{3}) = \frac{4}{3}\sqrt{3}\pi + 1 \approx 8.2552$

$f(\frac{5\pi}{3}) = \frac{5}{3}\sqrt{3}\pi - 1 \approx 8.0690$

$f''(x) = 2\cos x \stackrel{SET}{=} 0$
 $\Rightarrow x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$

$f(\frac{\pi}{2}) = \frac{\sqrt{3}}{2}\pi \approx 2.7207$

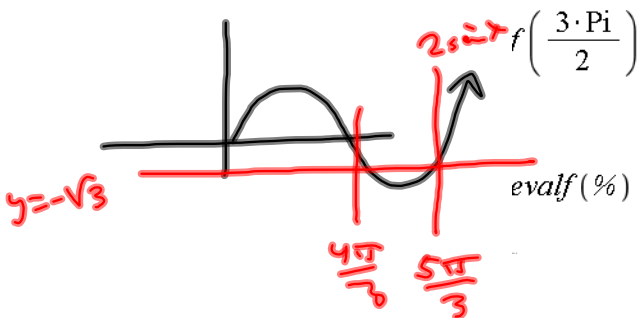
$f(\frac{3\pi}{2}) = \frac{3}{2}\sqrt{3}\pi \approx 8.1621$

$$f\left(\frac{\pi}{2}\right)$$

$$\frac{1}{2}\sqrt{3}\pi$$

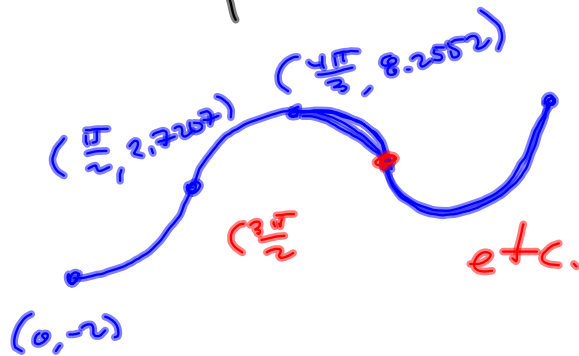
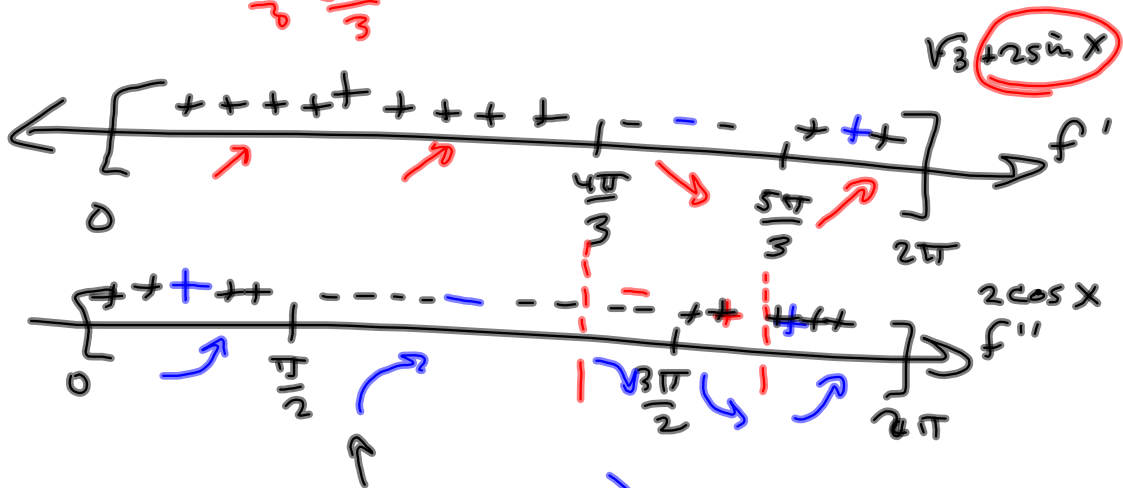
evalf(%)

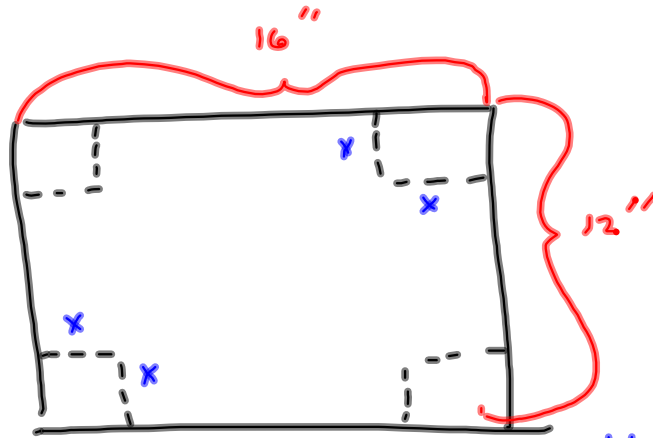
2.720699048



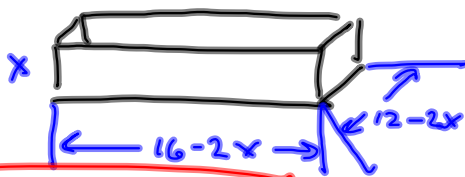
$$\frac{3}{2}\sqrt{3}\pi$$

8.162097142





Maximize Volume of the box



$$\begin{aligned}
 V &= x(16-2x)(12-2x) \\
 &= x(2x-16)(2x-12) \\
 &= 4x(x-8)(x-6)
 \end{aligned}$$

$$\begin{array}{r}
 196 \\
 2 \\
 \hline
 192
 \end{array}$$

$$\begin{aligned}
 \frac{dV}{dx} &= 4(x-8)(x-6) + 4x(1)(x-6) + 4x(x-8)(1) \\
 &= 4(x^2 - 14x + 48) + 4x^2 - 24x + 4x^2 - 32x \\
 &= 4x^2 - 56x + 192 + 8x^2 - 56x \\
 &= 12x^2 - 112x + 192 \\
 &= 4(3x^2 - 28x + 48) \stackrel{SET}{=} 0
 \end{aligned}$$

$$\begin{array}{r}
 2 \overline{) 112} \\
 \underline{224} \\
 2 \overline{) 56} \\
 \underline{2112} \\
 2 \overline{) 28} \\
 \underline{214} \\
 7
 \end{array}$$

$\rightarrow x \approx 7.07637$
 or 2.26297 inches

$$\begin{array}{r}
 2 \overline{) 192} \\
 \underline{236} \\
 2 \overline{) 56} \\
 \underline{2112} \\
 2 \overline{) 24} \\
 \underline{214} \\
 2 \overline{) 12} \\
 \underline{26} \\
 3
 \end{array}$$

solve($3 \cdot x^2 - 28 \cdot x + 48 = 0, x$)

$$\frac{14}{3} + \frac{2}{3} \sqrt{13}, \frac{14}{3} - \frac{2}{3} \sqrt{13}$$

evalf(%)

7.070367517, 2.262965817

$$V = x(16-2x)(2-2x)$$

$$= x(2x-16)(2x-12)$$

$$= 4x(x-8)(x-6)$$

$$= 4x(x^2 - 14x + 48)$$

$$= 4x^3 - 56x^2 + 192x = V$$

$$x \approx 7.07037$$

$$\text{or } 2.26297 \text{ inches}$$



$$\frac{dV}{dx} = 12x^2 - 112x + 192$$

$$\frac{d^2V}{dx^2} = 24x - 112 = V'(x)$$

2nd
Derivative
Test

$$V''(7.07) \approx 169.68 - 112 > 0 \quad \ddot{u}$$

$$V''(2.26) \approx 54.24 - 112 < 0 \quad \text{MIN} \quad \ddot{u}$$

MAX