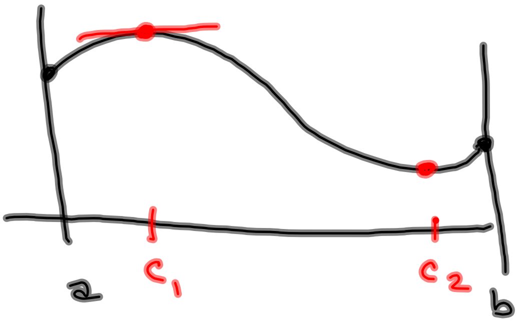
§4.1

Absolute max/min on a domain, $\mathcal{D}$.

This occurs at $x=c$ if

(i) max : $f(c) \geq f(x)$ $\forall$ $x \in \mathcal{D}$

(ii) min : $f(c) \leq f(x)$ " "



$(c_1, f(c_1))$ is an extreme point

Book calls 'c' the point. I think of the point as $(c_1, f(c_1))$

c_1 is the x-value

$f(c_1)$ is the maximum value

$(c_1, f_1(c))$ is the absolute maximum point.

$(c_2, f(c_2))$ is absolute minimum pt.

By book definition, every pt on the plateau is a maximum pt.

EVT - Extreme Value Theorem

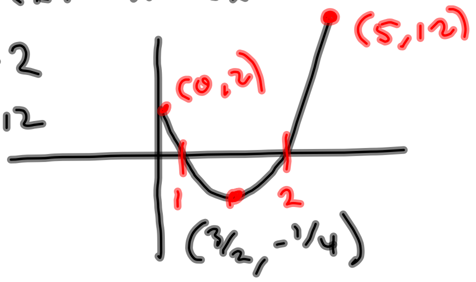
Any continuous function on a closed interval has (achieves) an absolute max & an absolute min in that interval.

$(x-1)(x-2)$

$f(x) = x^2-3x+2$ on $[a,b] = [0,5]$

$f(0)=2$

$f(5)=12$



$f(\frac{3}{2}) = -\frac{1}{4}$

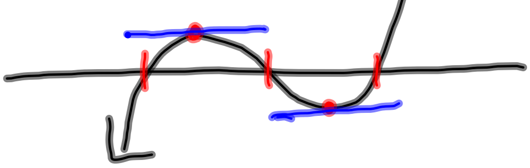
$$\boxed{x^2-3x+2}$$
$$= x^2-3x+\left(\tfrac{3}{2}\right)^2-\tfrac{9}{4}+\tfrac{8}{4}$$
$$= \boxed{\left(x-\tfrac{3}{2}\right)^2-\tfrac{1}{4}}$$

Abs Max: $y=12$ @ $x=5$

" " pt: $(5,12)$

Abs. Min: $y=-\frac{1}{4}$ @ $x=\frac{3}{2}$

" " pt: $(3/2, -1/4)$



$$y' = 3x^2 - 4x + 1$$

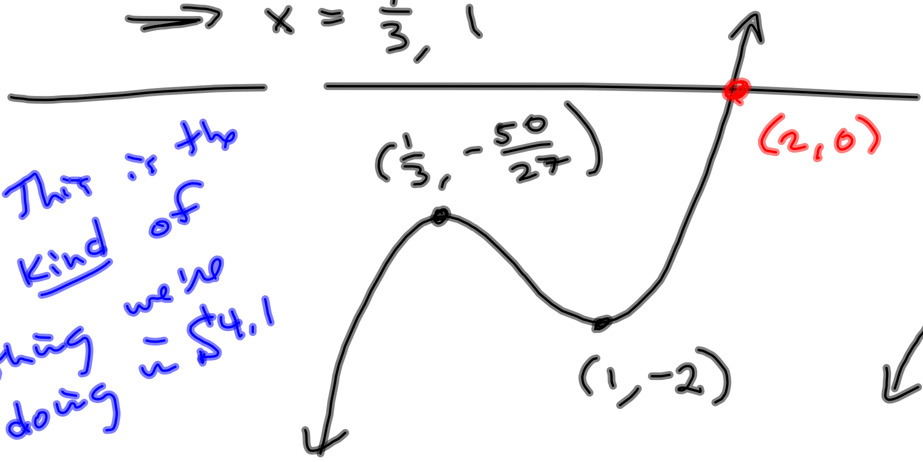
$$y = \frac{3x^3}{3} - \frac{4x^2}{2} + x$$

$$y = +x^3 - 2x^2 + x - 2 = x^2(x-2) + 1(x-2) = \underline{(x^2+1)}(x-2$$

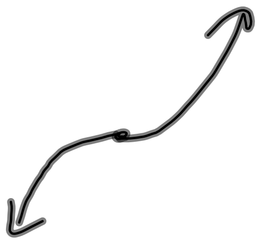
$$f(x) = x^3 - 2x^2 + x - 2 \quad \text{Sketch it.}$$

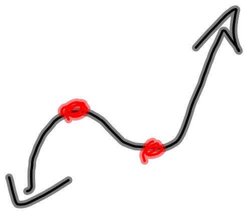
$$f'(x) = 3x^2 - 4x + 1 = (3x-1)(x-1) \overset{\text{SET}}{=} 0$$

$$\Longrightarrow x = \frac{1}{3}, 1$$



This is the kind of thing we're doing in §4.1





$$f\left(\tfrac{1}{3}\right) = \left(\tfrac{1}{3}\right)^3 - 2\left(\tfrac{1}{3}\right)^2 + \tfrac{1}{3} - 2$$

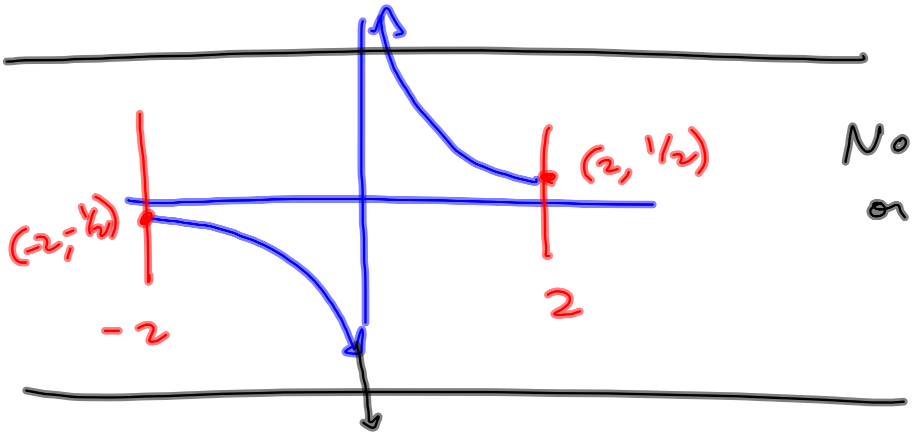
$$= \frac{1}{27} - \frac{2}{9} + \frac{1}{3} - 2$$

$$= \frac{1 - 6 + 9 - 54}{27}$$

$$= \frac{-50}{27}$$

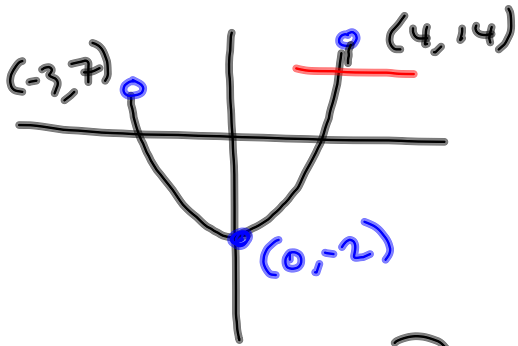
Non-examples of EVT.

$\frac{1}{x}$ on $[-2,2]$ has no abs. max/min



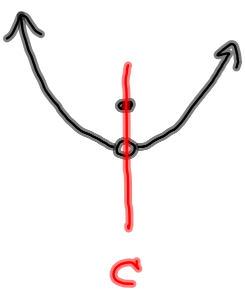
No ceiling or floor.

$f(x) = x^2 - 2$ on $(-3,4)$



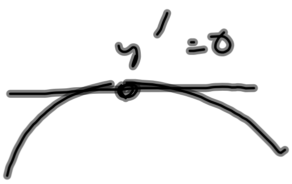
Abs min @ $(0,-2)$

$y=14$ is the $\boxed{\text{least upper bound,}}$ But $f(x)$ never achieves $y=14$.



Doesn't achieve a min

THIS function has a Local Max @ $(c, f(c))$



CRITICAL point (value):

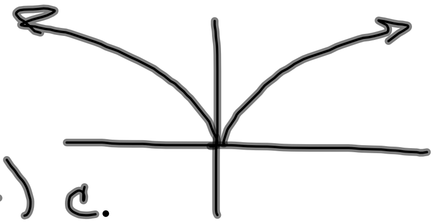
$f'(c) = 0$ OR $f'(c) \nexists$

Hilltop
valley bottom

cusp

$|x|$

$y = x^{2/3}$

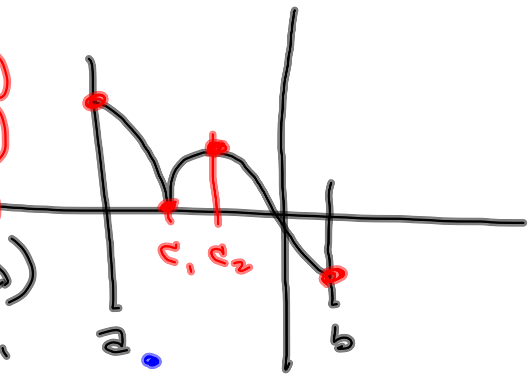


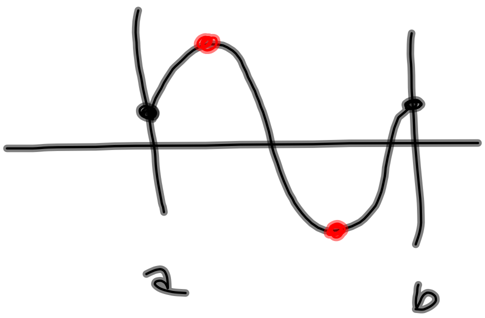
Method for extreme values on [a,b]:

① f(c) at critical points (values) c.
AND f(a) and f(b)

Abs Max @ $(a, f(a))$
Abs Min @ $(b, f(b))$
$f' \nexists$ Local Min @ $(c_1, f(c_1))$
$f' = 0$ Local Max @ $(c_2, f(c_2))$

② Take the biggest/smallest.





5' #s 1, 4, 7, 10 – 14, 17, 18 More to come.