9 $\frac{dy}{dt} = -2$, FIND $\frac{dx}{dt}$

10 $r + s^2 + v^3 = 12$, given $\frac{dr}{dt} = 4$, $\frac{ds}{dt} = -3$

Find $\left.\frac{dv}{dt}\right|_{\substack{r=3 \\ s=1}}$

Find v:

$3 + 1^2 + v^3 = 12$

$v^3 = 8$

$v = 2$

$\frac{d}{dt}\left[r + s^2 + v^3 = 12\right] \Longrightarrow$

$\frac{dr}{dt} + 2s\frac{ds}{dt} + 3v^2\frac{dv}{dt} = 0$

$4 + 2(1)(-3) + 3v^2\frac{dv}{dt} = 0$

$4 - 6 + 3v^2\frac{dv}{dt} = 0$

$-2 + 3v^2\frac{dv}{dt} = 0$

$3v^2\frac{dv}{dt} = 2$

$\boxed{\frac{dv}{dt} = \frac{2}{3v^2}} = \frac{2}{3(2)^2} = \frac{2}{12} = \frac{1}{6}$

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If $f(x)$ has derivatives of all orders, then the Taylor's series for $f(x)$ is

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k = T_a(x)$$

$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(k)}(a)}{k!}(x-a)^k + \cdots$$

$$f(x) \approx \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

sin x on a calculator.

$T_0(x)$

$f(x) = \sin x \quad \sin(0) = 0$

$f'(x) = \cos x \quad \cos(0) = 1$

$f''(x) = -\sin x \quad -\sin(0) = 0$ ✓

$f'''(x) = -\cos x \quad -\cos(0) = -1$

3rd degree Taylor Poly!

$$\sum_{k=0}^{3} \frac{f^{(k)}(a)}{k!}(x-a)^k \quad 0 + \frac{1}{1}(x-0)^1 + 0 + \frac{(-1)}{3!}(x-0)^3 + \cdots$$

$$= x - \frac{x^3}{3!} = T(x)$$

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$T\left(\frac{\pi}{6}\right) = \frac{\pi}{6} - \frac{\left(\frac{\pi}{6}\right)^3}{6} \approx .499$$

5th

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$

Calculators do powers, products, & sums FAST.

3.1, 3.2 Last time we do

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{z\to x} \frac{f(z)-f(x)}{z-x}$$

"By the Definition" is the only time you have to do it this way.

Otherwise:

Power Rule $\frac{d}{dx}[x^n] = n x^{n-1}$

Product Rule $\frac{d}{dx}[fg] = \frac{df}{dx} g + f \frac{dg}{dx}$
$= f'g + fg'$

Quotient Rule $\frac{d}{dx}\left[\frac{f}{g}\right] = \dfrac{\frac{df}{dx} g - f\frac{dg}{dx}}{g^2}$
$= \dfrac{f'g - fg'}{g^2}$

Chain Rule $\frac{d}{dx}[f(g(x))] = \frac{df}{dg} \cdot \frac{dg}{dx}$ Cool.
$= f'(g(x))\, g'(x)$, which I don't like

$$\frac{d}{dx}\left[\left(\sin\left(\left(\tan\left((x^2-5x)^{2/3}\right)\right)^{3/4}\right)\right)^4\right]$$

$$= 4\sin\left(\left(\tan\left((x^2-5x)^{2/3}\right)\right)^{3/4}\right)^3 \cos\left(\left(\tan\left((x^2-5x)^{2/3}\right)\right)^{3/4}\right) \cdot$$

$$\frac{3}{4}\left(\tan\left((x^2-5x)^{2/3}\right)\right)^{-1/4}\left(\sec^2\left((x^2-5x)^{2/3}\right)\right) \cdot$$

$$\frac{2}{3}(x^2-5x)^{-1/3}(2x-5)$$

$$\frac{d}{dx}\left[\left(\sin(x^2-5x)\right)^{3/4}\right] =$$

$$\frac{3}{4}\left(\sin(x^2-5x)\right)^{-\frac{1}{4}}\left(\cos(x^2-5x)\right)(2x-5)$$

3.1 Find eq'n of tan. line @ (1,2) to

$f(x) = 2\sqrt{x} = 2x^{\frac{1}{2}}$

$f'(x) = x^{-\frac{1}{2}}$

$f'(1) = 1$

$y = 1(x-1) + 2$

$= f'(a)(x-a) + f(a)$

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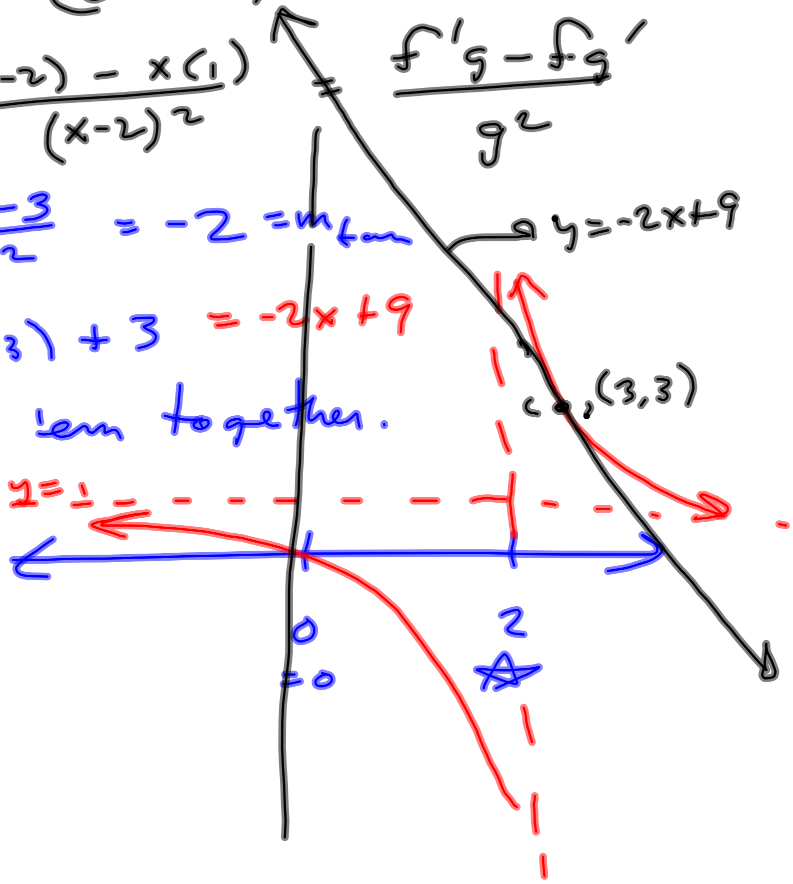
$h(x) = \frac{x}{x-2}$ @ $(3,3) = (a, f(a)) = (x_1, y_1)$

$h'(x) = \frac{1(x-2) - x(1)}{(x-2)^2} = \frac{f'g - fg'}{g^2}$

$h'(3) = \frac{1-3}{1^2} = -2 = m_{tan}$

$y = -2(x-3) + 3 = -2x + 9$

Graph 'em together.



$\frac{x}{x-2}$

$\lim_{x \to \infty} \frac{x/x}{x/x - 2/x} = \frac{1}{1 - 2/x}$

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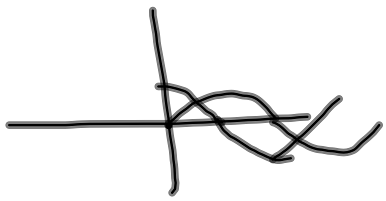
$\frac{d}{dx}\sin x$, $\cos x$, $\tan x$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{d}{dx}\left[(\sin x)^{-1}\right]$$

$$= -1(\sin x)^{-2}\cos x = -\frac{\cos x}{\sin^2 x} = -\csc x \cot x$$



$\sin x = \cos\left(x - \frac{\pi}{2}\right)$

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Find when $m_{\tan} = -\frac{3}{2}$, if possible.

$$y = \frac{x}{2} + \frac{1}{2x-4} = \frac{1}{2}x + (2x-4)^{-1}$$

$$y' = \frac{1}{2} - (2x-4)^{-2}(2)$$

$$= \frac{1}{2} - \frac{2}{(2x-4)^2} \overset{SET}{=} -\frac{3}{2}$$

$$\frac{1}{2} - \frac{\cancel{2}\,1}{\cancel{4}\,2\,(x-2)^2} = -\frac{3}{2}$$

$$\frac{x^2-4x+4-1}{2(x-2)^2} = \frac{-3(x-2)^2}{2(x-2)^2} = \frac{-3x^2+12x-12}{2(x-2)^2}$$

$$x^2-4x+3 = -3x^2+12x-12$$

$$4x^2-16x+15 = 0$$

$$4x^2-10x-6x+15 = 0$$

$$2x(2x-5)-3(2x-5) = 0$$

$$(2x-5)(2x-3) = 0$$

$$x \in \left\{\frac{5}{2}, \frac{3}{2}\right\}$$

$$(2x-4)^2 = (2(x-2))^2 = 4(x^2-4x+4)$$

$$= 4x^2-16x+16$$