$h=10 \Rightarrow \; = \frac{\pi}{4}D^2 \cdot 10 = \frac{5\pi}{2}D^2$ For later.

$V = \pi r^2 h = \pi\left(\frac{1}{2}D\right)^2 h = \frac{\pi}{4}D^2 h$

$\Delta V \approx dV = \frac{\pi}{2} D\, h\, dD$ Given $h=10$. No change in h

$\frac{dV}{dD} = \frac{\pi}{2}Dh + \frac{\pi}{4}D^2\frac{dh}{dD}$

$=0$. $h=$ constant.

$dV = \frac{\pi}{2}D(10)\,dD$

$dV = 5\pi D\, dD$

Want $|\Delta V| \approx |dV| = 5\pi D\, dD \overset{\text{want}}{\leq} .01V$

$\cancel{5\pi D}\, dD \leq .01\left(\frac{\cancel{5\pi}}{2}D^{\cancel{2}}\right)$

$\boxed{dD \leq \frac{.01}{2}D} = .005D$

$\frac{dD}{D} \leq .005 = 0.5\%$

So % error in diameter measurement must be $\leq .5\%$

I expected This to get a # for the allowable error in D's measurements

Not knowing D, I resorted to relative error $\frac{dD}{D}$

% error $\left(\frac{dD}{D}\right)(100\%)$

Oct 2-12:13 PM

3.9 #54

(a) $Q_a(x) = b_0 + b_1(x-a) + b_2(x-a)^2$

$f(x) = \frac{1}{1-x}$

(1) $Q_a(a) = f(a)$

$b_0 = f(a)$

$Q_a'(x) =$

$b_1 + 2b_2(x-a)$

$Q_a''(x) = 2b_2$

(2) $Q_a'(a) = \cancel{f'(a)}$

$Q'(x) = b_1 + 2b_2(x-a)'$

(3) $Q''(a) = f''(a)$

$2b_2 = f''(a)$

$b_2 = \frac{1}{2} f''(a)$

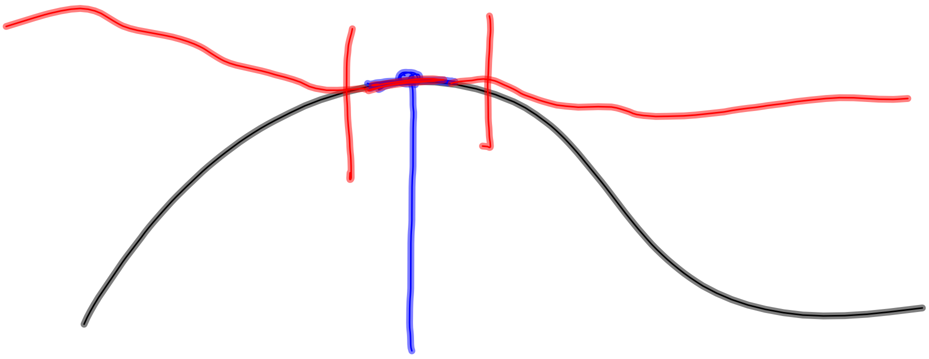
$Q_a(x) = b_0 + b_1(x-a) + b_2(x-a)^2$

$= b_0 + b_1 x - b_1 a + b_2(x^2 - 2ax + a^2)$

$= b_0 + b_1 x - b_1 a + b_2 x^2 - 2ab_2 x + b_2 a^2$

$Q_a'(x) = b_1 + 2b_2 x - 2ab_2$

$Q_a'(a) = b_1 + 2b_2 a - 2ab_2 = b_1 = f'(a)$



a

$$Q_a(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

Cubic:

$$C_a(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + b_3(x-a)^3$$

Want:

$$C_a'''(a) = f'''(a)$$

$c(x) = b_0 + b_1(x-a) + b_2(x-a)^2 + b_3(x-a)^3$ $\Rightarrow$ what will b_3 be?

$c'(x) = b_1 + 2b_2(x-a) + 3b_3(x-a)^2$

$c''(x) = 2b_2 + 2\cdot 3\, b_3(x-a)$

$c'''(x) = 2\cdot 3\, b_3$

$c'''(a) = f'''(a) \Longrightarrow$

$2\cdot 3\, b_3 = f'''(a) \Longrightarrow$

$$b_3 = \frac{1}{3\cdot 2} f'''(a)$$

$+ b_4(x-a)^4$

a': $4b_4(x-a)^3$

a'': $3\cdot 4\, b_4(x-a)^2$

a''': $2\cdot 3\cdot 4\, b_4(x-a)$

$c^{(4)}$: $2\cdot 3\cdot 4 = 4!$

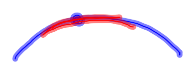
a''

10^{th} - degree approximation

$$f(x) \approx \frac{f(a)}{0!} + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2$$
$$+ \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(10)}}{10!}(x-a)^{10}$$

This is the 10^{th} TAYLOR POLYNOMIAL for $f(x)$.

Tangent Line is the 1^{st} -degree Taylor Polynomial.



(b) $Q(x) = b_0 + b_1(x-0) + b_2(x-0)^2$
$$= f(0) + f'(0)(x-0) + \frac{f''(0)}{2}(x-0)^2$$

$f(x) = \frac{1}{1-x}$ @ $x = 0 = a$

$f(x) = (1-x)^{-1}$

$f'(x) = -(1-x)^{-2}(-1) = (1-x)^{-2} = \frac{1}{(1-x)^2}$

$f''(x) = -2(1-x)^{-3}(-1) = \frac{2}{(1-x)^3}$

$f(0) = 1$

$f'(0) = 1$

$f''(0) = 2$

$Q_0(x) = 1 + 1(x-0) + \frac{2}{2}(x-0)^2$

$$\boxed{= 1 + x + x^2}$$

Geometric Series LOOK IT UP

$\boxed{1 + r + r^2} + r^3 + \cdots + r^n + \cdots = \frac{1}{1-r}$

$$S_n = 1 + \cancel{r} + \cancel{r^2} + \cancel{\cdots} + \cancel{r^n}$$
$$-rS_n = -(\cancel{r} + \cancel{r^2} + \cancel{r^3} + \cancel{\cdots} + \cancel{r^n} + r^{n+1})$$

$S_n - rS_n = 1 - r^{n+1}$

$S_n(1-r) = 1 - r^{n+1}$

$S_n = \frac{1-r^{n+1}}{1-r}$ what if $0 < r < 1$

$S_n \xrightarrow{n \to \infty} \frac{1-0}{1-r} = \frac{1}{1-r}$ is the formula for a convergent geometric series.